Advances in Water Resources 32 (2009) 205–212

Contents lists available at ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

Network flow modeling via lattice-Boltzmann based channel conductance

Yelena Sholokhova *, Daesang Kim, W. Brent Lindquist

Department of Applied Mathematics and Statistics, Stony Brook University, Stony Brook, NY 11794-3600, USA

ARTICLE INFO

Article history:
Received 22 July 2008
Received in revised form 20 October 2008
Accepted 21 October 2008
Available online 8 November 2008

Keywords:
Network flow models
Absolute permeability
Single-phase flow
Hydraulic conductance
Lattice-Boltzmann

ABSTRACT

Lattice-Boltzmann (LB) computations of single phase, pore-to-pore conductance are compared to models in which such conductances are computed via standard pore body–channel–pore body series resistance (SR), with the conductance of each individual element (pore body, channel) based on geometric shape factor measurements. The LB computations, based upon actual channel geometry derived from X-ray computed tomographic imagery, reveal that the variation in conductance for channels having similar shape factor is much larger than is adequately captured by the geometric models. Fits to the dependence of median value of conductance versus shape factor from the LB-based computations show a power law dependence of higher power than that predicted by the geometric models. We introduce two network flow models based upon the LB conductance computations: one model is based upon LB computations for each pore-to-pore connection; the second is based upon a power law fit to the relationship between computed conductance and throat shape factor. Bulk absolute permeabilities for Fontainebleau sandstone images are computed using the SR-based network models and the two LB-based models. Both LB-based network models produce bulk absolute permeability values that fit published data more accurately than the SR-based models.

1. Introduction

Network flow models [1–20] and, more recently, lattice-Boltzmann models [21–24] are the dominant numerical tools for simulating flow at the pore-scale level. Initially used for single-phase computation, they are being utilized for increasingly complex flow simulations – multiphase, multicomponent, and reactive. In the progression to increasingly complex simulation, it is vital to continually assess the fundamentals of these numerical algorithms. In particular, network flow models capture a simplified description of a pore network, relying on computations based upon geometric assumptions to compute dynamic network parameters. A key advance in two-phase network flow modeling was enabled by the development of the ability to analytically compute entrance pressures for a tube of constant cross-sectional shape that is either triangular [11,25] or regular polygonal [26]. Recently, this analytic ability has since been extended to polygonal cross-sections of arbitrary convex shape [27]. These advances have allowed modeling a degree of irregularity in channel cross-sectional shape that was not available in earlier models which required circular or square cross-sectional shape. Of particular benefit from this is the controlled modeling of film flow [11,15].

A fundamental quantity required for any network flow model is the conductance governing the flow between any two connected pore neighbors in the network. For rectangular and elliptical cross-sections, the effective conductance governing incompressible, low-Reynolds number, Navier–Stokes, single-phase flow can be computed analytically [15]. For a regular polygon or irregular triangle cross-section shape, numerical computations are required [11] to compute conductances. These computations show that a monotonic relation exists between the dimensionless conductance and a dimensionless shape factor [25] that characterizes each of these shapes. This monotonic relation can be pre-computed and, in the case of numerically computed relations, fit to an accurate analytic form which is then used in the network flow model. These shape factor–conductance relationships perfectly capture the spirit of network flow models – namely to extract critical, relatively simple, experimentally accessible measures of the pore network geometry that can be used to develop accurate network flow models.

Øren et al. [11] and Patzek [15] have described network flow models based upon these shape factor–conductance relationships. These models are standard, in that pore-to-pore conductance is computed via a pore body–channel–pore body series resistance (SR), with the conductance of each individual element (pore body, channel) based upon measurements of shape factor. The cross-sectional shapes of the network elements (pore bodies, channels) are determined by fitting to shape factor data taken from simulated or real media. While triangular shapes have been a desired feature of these models, due to limitations on the range of shape factors available for triangles, cross-sections in these models are

* Corresponding author.
E-mail address: sholy@ams.sunysb.edu (Y. Sholokhova).

0309-1708/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.advwatres.2008.10.016

augmented with rectangular and elliptical cross-sections to model the broader range of shape factors observed in real porous media. In Section 3, in the context of single-phase flow, we review these shape factor–conductance relationships and the network flow models based upon these relationships.

The conductance estimates based upon shape factor work surprisingly well in spite of the introduction of potential "peculiarities" in the geometry of the pores. In the triangular cross-section models, the conductance computation for each pore–channel–pore connection assumes constant cross-sections of triangular shape separately for each of the two types of elements (pore, channel) forming the connection. Thus pores and channels are effectively modeled as triangular prisms. While this is conceptually acceptable for channels, it is difficult to reconcile the geometry of a pore which has the same triangular prism shape in *every* direction to which it connects to a neighboring pore.

For modeling flow through real media, use of X-ray computed tomography (XCT) to produce 3D image sets of rock samples, coupled with analysis software such as the 3DMA-Rock package [28], allows relatively rapid characterization of rock samples at the pore level, and the opportunity to incorporate greater geometric detail into network flow model descriptions. In Section 2, we summarize the XCT analysis of four Fontainebleau data sets, ranging from 7.5% to 22% porosity, used in this study.

Of particular interest to us is the detailed geometry of the connection between any two neighboring pores and the effect of approximating this geometry when computing effective pore-to-pore conductance. We compare three methods of computing pore-to-pore conductance. The first (Section 3) is a series resistance model utilizing conductances estimated from shape factors (G) and triangular/rectangular/elliptical geometries. We refer to this method as SRG. In the second method (Section 4), we utilize a lattice-Boltzmann (LB) model to compute an accurate approximation to Navier–Stokes flow through each isolated pore-to-pore connection in the Fontainebleau data sets and determine a connection-specific conductance. We refer to this method as LBC. This method, which involves running an LB computation separately on each channel is CPU intensive. We therefore consider a third method where pore-to-pore conductances are computed using an analytic expression obtained from fit between LBC computed conductances and throat shape factors, G. We refer to this third method of computing pore-to-pore conductances as LBG.

In Section 5 we compare the results for computing the effective (i.e. bulk) absolute permeability for the four sandstone networks using network flow models based upon the three methods for estimating pore-to-pore conductance. The first network flow model (NF-SRG), which closely follows the work of Øren et al. [11] and Patzek [15], utilizes the SRG method for estimating pore-to-pore conductance. We also investigate the sensitivity of the NF-SRG model to the weighting of the elements in the series resistance of each pore-to-pore connection. The second model (NF-LBC) directly uses the LBC computed conductances. This model eliminates the need for both series resistance and cross-sectional geometry approximation in the computation of the pore-to-pore conductances. The third model (NF-LBG) uses the LBG method for computing pore-to-pore conductance. Use of NF-LBG eliminates the need for series resistance approximation. All models are validated against published measurements [29].

2. Fontainebleau sandstone data

The network models constructed in this paper are based upon an analysis of Fontainebleau sandstone samples with porosities of 7.5%, 13%, 18% and 22%. The XCT images, digitized at 5.7 μm voxel size, were taken of cores 5 mm in diameter at the X2B beam line at the National Synchrotron Light Source at Brookhaven National Laboratory. From each image, a $550 \times 550 \times 511$ voxel (28.63 mm^3) rectangular subvolume was analyzed. Previous analyses on these samples [30] has shown that volumes of this size produce statistically reproducible results with respect to characterization of geometric parameters of the pore network. Fig. 1 shows the middle slice from the analyzed region for the 22% porosity image.

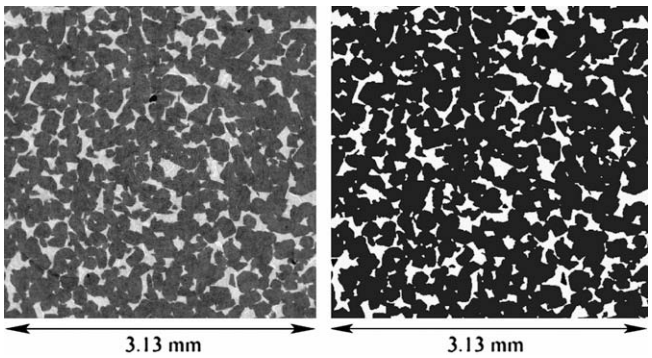

Fig. 1. A single slice from the XCT image of the 22% Fontainebleau sandstone sample analyzed by 3DMA-Rock (left). The same slice after segmentation of the image via the indicator kriging algorithm in 3DMA-Rock (right).

Geometrical characterization of the pore network was performed using the software package 3DMA (see Stony Brook University technical report, SUNYSB-AMS-99-20 at ftp://ftp.ams.sunysb.edu/pub/papers/1999/susb99.20.pdf and the 3DMA-Rock home page at http://www.ams.sunysb.edu/~lindquis/3dma/3dma_rock/3dma_rock.html). The main algorithmic procedures used to extract pore network information were: image segmentation by indicator kriging [31] to partition the image into void and grain phases (see Fig. 1); construction of the medial axis [32] and extraction of its percolating backbone to provide a search path through the pore network; determination of throat locations (cross-sections of locally minimum area) in channels connecting pore bodies [33–35]; and identification of a pore body/throat network, where every pore body is cross-indexed with its connecting throats and adjoining pores [35]. It is important to note that our characterization is a pore body/throat network and not a pore body/channel network. Each throat is constructed as a surface of contact between two pore bodies. These surfaces are triangulated, not necessarily planar, polygons. The surfaces occupy no volume; all the volume of the pore network is in the pore bodies. For each throat surface we compute its location (point where it intersects the medial axis), area, perimeter length and two principal diameters. For each pore body we compute its center of mass (CoM) location, volume, surface area (via the method of Marching Cubes [36]), three principal diameters, and coordination number (number of connecting pore bodies). The CoM to CoM distance along the connecting medial axis is computed between every pair of connected pore body neighbors, as well as the distance between a pore body CoM and the location of each throat separating the pore body from its neighbors.

3. Hydraulic conductance based upon series resistance: the NF-SRG models

The single-phase flow rate between neighboring pore bodies i and j in a network flow model is described by Darcy's law

$$Q_{ij} = \frac{g_{ij}}{l_{ij}}(p_i - p_j), \tag{1}$$

where Q_{ij} is the volumetric flow rate $[L^3/T]$, g_{ij} is the effective fluid conductance $[L^5T/M]$, l_{ij} is the distance between the pore centers, and p_i is the pressure (at the CoM) of pore i. A standard computation