Available online at www.sciencedirect.com
ScienceDirect
Advances in Water Resources 31 (2008) 599–608
Advances in Water Resources
www.elsevier.com/locate/advwatres

Flow based oversampling technique for multiscale finite element methods

J. Chu [a], Y. Efendiev [b,*], V. Ginting [c], T.Y. Hou [a]

[a] Applied Mathematics, Caltech, Pasadena, CA 91125, United States
[b] Department of Mathematics, Texas A&M University, College Station, TX 77843-3368, United States
[c] Department of Mathematics, University of Wyoming, Laramie, WY 82071, United States

Received 11 June 2007; received in revised form 6 November 2007; accepted 15 November 2007
Available online 5 December 2007

Abstract

Oversampling techniques are often used in porous media simulations to achieve high accuracy in multiscale simulations. These methods reduce the effect of artificial boundary conditions that are imposed in computing local quantities, such as upscaled permeabilities or basis functions. In the problems without scale separation and strong non-local effects, the oversampling region is taken to be the entire domain. The basis functions are computed using single-phase flow solutions which are further used in dynamic two-phase simulations. The standard oversampling approaches employ generic global boundary conditions which are not associated with actual flow boundary conditions. In this paper, we propose a flow based oversampling method where the actual two-phase flow boundary conditions are used in constructing oversampling auxiliary functions. Our numerical results show that the flow based oversampling approach is several times more accurate than the standard oversampling method. We provide partial theoretical explanation for these numerical observations.

Keywords: Multiscale; Finite volume; Oversampling; Upscaling; Two-phase flow

1. Introduction

The high degree of variability and multiscale nature of formation properties such as permeability pose significant challenges for subsurface flow modeling. Geological characterizations that capture these effects are typically developed at scales that are too fine for direct flow simulation, so techniques are required to enable the solution of flow problems in practice. Upscaling procedures have been commonly applied for this purpose and are effective in many cases (see [28,26,19] for reviews and discussion). More recently, a number of multiscale finite element (e.g., [20,10,3,1,2,17]) and finite volume [22,23] approaches have been developed and successfully applied for problems of this type.

Our purpose in this paper is to propose a new oversampling strategy in constructing multiscale basis functions within the framework of multiscale finite element method (MsFEM). The MsFEM was first introduced in [20]. Its main idea is to incorporate the small-scale information into finite element basis functions and capture their effect on the large-scale via finite element computations. There are a number of multiscale numerical methods (or framework) with similar general objective, such as generalized finite element methods [5], residual free bubbles [27], variational multiscale method [21], multiscale finite element method (MsFEM) [20], two-scale finite element methods [24], two-scale conservative subgrid approaches [3], heterogeneous multiscale method (HMM) [16], and multiscale mortar methods [4]. We remark that special basis functions in finite element methods have been used earlier in [6]. Multiscale finite element methodology has been modified and successfully applied to two-phase flow simulations in [22,23] and later in [10,1].

* Corresponding author. Tel.: +1 979 845 1972.
E-mail address: efendiev@math.tamu.edu (Y. Efendiev).

0309-1708/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.advwatres.2007.11.005

Most multiscale methods presented to date have applied local calculations for the determination of basis functions (or, in the case of variational multiscale methods [3], subgrid integrals). Though effective in many cases, the accuracy of these local calculations may deteriorate for problems in which global effects are crucial. The importance of global information has been illustrated within the context of upscaling procedures in recent investigations [9,8].

These studies have shown that the use of global information in the calculation of the upscaled parameters can significantly improve the accuracy of the resulting coarse model.

In this paper, we propose a flow based oversampling method. The main idea of oversampling techniques is to use solutions of the underlying single-phase flow equation in larger domains for computing the basis functions. These basis functions are used in the two-phase flow simulations with varying (dynamic) mobility. Oversampling techniques reduce the effect of artificial boundary conditions that are often imposed in computing local quantities, such as upscaled permeabilities or basis functions. When there is no scale separation, the oversampling region is taken to be the entire domain. Typically, generic boundary conditions are used to compute the auxiliary oversampling functions. These boundary conditions do not reflect the actual two-phase flow boundary conditions which can have large effect in the simulations. In particular, when two-phase flow boundary conditions contain some type of singularities, the single-phase flow solutions obtained using generic boundary conditions are not sufficient to represent these effects. For this reason, one needs to incorporate the actual two-phase flow boundary conditions. In the proposed flow based oversampling method, we take one (or more) auxiliary oversampling functions to be the solution of single-phase flow equations with original (two-phase flow) boundary information. We present a partial analysis which demonstrates the importance of using the actual boundary conditions. Moreover, our analysis explains when one needs to use the actual two-phase flow boundary conditions which is associated to the "singularity" in the boundary conditions of two-phase flows.

To illustrate the performance of this new strategy, we present several representative numerical results. In particular, comparison between the flow based and standard oversampling is made for typical two-phase flow and transport simulations. In our numerical results, we use the permeability fields from the SPE comparative project [12]. These permeability fields are channelized and difficult to upscale. In particular, due to channelized nature of these permeability fields, the non-local effects are important and, often, some type of limited global information is used in multiscale simulations (e.g., [17,15]). In our simulations, we test various viscosity ratios and compare integrated quantities, such as oil production rate and total flow rate, as well as the saturation errors at some time instances. In all cases, we observe that the flow based oversampling methods are more accurate and, in almost all the cases we consider, it gives several orders of improvement.

The paper is organized in the following way. In the next section we give some preliminaries explaining the two-phase flow fine-scale model and the multiscale finite volume element method (MsFVEM). In Section 3, we present the flow based oversampling approach and analysis. Finally, in Section 4, the numerical results are presented.

2. Preliminaries

We consider two-phase flow in a reservoir Ω under the assumption that the displacement is dominated by viscous effects; i.e., we neglect the effects of gravity, compressibility, and capillary pressure. Porosity is assumed to be constant. In this flow problem, the two phases are water and oil, designated by subscripts w and o, respectively. We write Darcy's law, with all quantities dimensionless, for each phase as follows:

$$\mathbf{v}_j = -\frac{k_{rj}(S)}{\mu_j}\mathbf{k}\cdot\nabla p, \tag{2.1}$$

where $\mathbf{v}_j$ is the phase velocity, $\mathbf{k}$ is the permeability tensor, k_{rj} is the relative permeability to phase j ($j = o, w$), S is the water saturation (volume fraction), p is pressure and μ_j is the viscosity of phase j ($j = o, w$). In this work, a single set of relative permeability curves is used and $\mathbf{k}$ is assumed to be a diagonal tensor. Combining Darcy's law with a statement of conservation of mass allows us to express the governing equations in terms of the so-called pressure and saturation equations:

$$\nabla\cdot(\lambda(S)\mathbf{k}\cdot\nabla p) = h, \tag{2.2}$$

$$\frac{\partial S}{\partial t} + \mathbf{v}\cdot\nabla f(S) = h_{\mathrm{w}}, \tag{2.3}$$

where λ is the total mobility, f is the fractional flow of water, $h = h_{\mathrm{w}} + h_{\mathrm{o}}$ is a source/sink term and $\mathbf{v}$ is the total velocity, which are respectively given by:

$$\lambda(S) = \frac{k_{\mathrm{rw}}(S)}{\mu_{\mathrm{w}}} + \frac{k_{\mathrm{ro}}(S)}{\mu_{\mathrm{o}}}, \quad f(S) = \frac{k_{\mathrm{rw}}(S)/\mu_{\mathrm{w}}}{k_{\mathrm{rw}}(S)/\mu_{\mathrm{w}} + k_{\mathrm{ro}}(S)/\mu_{\mathrm{o}}}, \tag{2.4}$$

$$\mathbf{v} = \mathbf{v}_{\mathrm{w}} + \mathbf{v}_{\mathrm{o}} = -\lambda(S)\mathbf{k}\cdot\nabla p. \tag{2.5}$$

The above descriptions are referred to as the fine model of the two-phase flow problem. Typical boundary conditions for (2.2) considered in this paper are fixed pressure at some portions of the boundary and no-flow on the rest of the boundary. For the saturation Eq. (2.3), we impose $S = 1$ on the inflow boundaries. For simplicity, in further analysis we will assume that $h_{\mathrm{w}} = h_{\mathrm{o}} = 0$ so that $h = 0$.

The upscaling of two-phase flow systems is discussed by many authors [11,7,14]. In most upscaling procedures, the coarse-scale pressure equation is of the same form as the fine-scale Eq. (2.2), but with an equivalent grid block permeability tensor $\mathbf{k}^*$ replacing $\mathbf{k}$. For a given coarse-scale grid block, the tensor $\mathbf{k}^*$ is generally computed through