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ScienceDirect
Advances in Water Resources 30 (2007) 1746–1755
Advances in Water Resources
www.elsevier.com/locate/advwatres

# Tide-induced groundwater head fluctuation in coastal multi-layered aquifer systems with a submarine outlet-capping

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Received 24 October 2006; received in revised form 16 January 2007; accepted 28 January 2007
Available online 3 February 2007

**Abstract**

This paper considered the tide-induced head fluctuations in two coastal multi-layered aquifer systems. Model I comprises two semi-permeable layers and a confined aquifer between them. Model II is a four-layered aquifer system including an unconfined aquifer, an upper semi-permeable layer, a confined aquifer and a lower semi-permeable layer. In each model, the submarine outlet of the confined aquifer is covered with a skin layer (“outlet-capping”). Analytical solutions of the two models are derived. In both models, leakages of the semi-permeable layers decrease the tidal head fluctuations. The outlet-capping reduces the aquifer’s head fluctuation by a constant factor and shifts the phase by a positive constant. The solution to Model II explains the inconsistency between the relatively small lag time and the strong amplitude damping effect of the tidal head fluctuations reported by Trefry and Johnston [Ground Water 1998;36:427–33] near the Port Adelaide River, Australia.


*Keywords:* Tide; Coastal aquifer system; Elastic storage; Leakage; Outlet-capping; Analytical solution; Peaty layer; Aquifer parameter estimation; Least-squares method; Tidal head fluctuation

## 1. Introduction

The sea level variations have significant impacts on the groundwater flow in many coastal areas. Tidal fluctuation is one of the most important driving forces. The hydraulic head or water table in a coastal aquifer system usually fluctuates in response to tidal fluctuations. Many analytical researches have been conducted in this field since the middle period of the last century. For example, Jacob [2] provided the analytical solution of tide-induced groundwater level fluctuation in a single coastal confined aquifer; van der Kamp [14] considered a coastal aquifer which extends under the sea infinitely; Li and Chen [5] investigated a coastal aquifer extending under the sea for a finite offshore length; Sun [11] developed an analytical solution in an estuary using a two-dimensional tidal loading boundary condition. The main limitation of these previous studies is that they only considered a single coastal aquifer configuration. In reality, most of the coastal aquifer systems are multi-layered, there is usually one or more confined aquifers and the semi-permeable layer(s) between them. Jiao and Tang [4] considered a coastal multi-layered aquifer system consisting of a confined aquifer, an unconfined aquifer, and a leaky layer between them. They ignored the leaky layer’s elastic storage. Li and Jiao [7] extended the result of Jiao and Tang [4] by taking the leaky layer’s elastic storage into account. Jeng et al. [3] presented a complete analytical solution to describe the tidal wave propagation and interference in the unconfined and confined aquifers separated by a thin, no-storativity leaky layer. Li and Jiao [8] considered a

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0309-1708/$ - see front matter 
doi:10.1016/j.advwatres.2007.01.003

coastal aquifer system including two aquifers separated by a semi-permeable layer, they improved previous work in that both the effects of the leaky layer's elastic storage and the tidal wave interference between two aquifers were considered.

The semi-permeable layer's elastic storage cannot be neglected when it is thick and has large specific storage, and may be even more important than that of the confined aquifer sometimes. According to various pumping test data available [7], if the semi-permeable layer is composed of thick, soft sedimentary materials, its elastic storage is much greater than that of the main aquifer. Most previous studies assumed that the coastal aquifer's submarine outlet is directly connected to the seawater. The reality, however, is not always that case. Li and Chen [5,6] mentioned that the submarine outlet of a coastal aquifer may usually be covered by a thin silt-layer (outlet-capping). Li et al. [9] modeled this layer but their study only considered a single coastal aquifer.

This paper investigates the joint actions of the outlet-capping and the leakage and elastic storage of the semi-permeable layer in two commonplace coastal multi-layered aquifer systems. Model I consists of two semi-permeable layers and a confined aquifer between them. Model II is a four-layered aquifer system including an unconfined aquifer, an upper semi-permeable layer, a confined aquifer and a lower semi-permeable layer. In each model, the confined aquifer has a outlet-capping on the sea-land boundary at the coastline. Mathematical models are given for describing the two aquifer systems. Exact analytical solutions to the models are derived. The solutions are compared with existing solutions by previous researchers.

Trefry and Johnston [13] analyzed the pumping test data in a tidally influenced aquifer near the Port Adelaide River in Australia. Standard transient analyses based on the well-known Theis's solution were first performed to the tidally corrected drawdown curves, yielding values of approximately 8–10 m/d for the hydraulic conductivity $K$ and 0.002 for the storativity $S$. Then the $S$-value and the Townley [12] solution were used to estimate the $K$-value based on the tidal effects. While the $K$-values (3–7 m/d) estimated from the tidal attenuation coefficients are roughly in line with the results of the pumping test analysis, those estimated from the tidal time lags (12–47 m/d) differ significantly from the pumping test analysis results, indicating a significant overestimation to the observed time-lags. Their numerical attempts using finite-element model to fit the tidal amplitudes and time lags in the observed drawdown curves by adjusting the storativity were also inconclusive. Based on these facts, they speculated that there may exist heterogeneities in the aquifer between the river bank and the test site.

In this paper, the new analytical solution is used to fit the observed tidal amplitudes and time lags in Trefry and Johnston [13]. The possibility of the existence of heterogeneities in the aquifer between the river bank and the test site is discussed, and the reason for the observed small time-lag is investigated.

## 2. Mathematical model and analytical solution

### 2.1. Mathematical model I

Model I is a coastal multi-layered aquifer system consisting of an upper and a lower semi-permeable layer and a confined aquifer between them (Fig. 1). The following assumptions will be used to the aquifer system:

1. The coastline is straight, all the layers are horizontal and extend landward infinitely, the bottom of the lower layer is impermeable.
2. Each of the three layers is homogeneous, isotropic and has constant thickness, the upper semi-permeable layer and the confined aquifer terminate at the coastline.
3. The flow is horizontal in the confined aquifer and vertical in the two semi-permeable layers [1,10].
4. The aquifer has a submarine outlet-capping, which may only cover the outlet of the middle confined aquifer as shown in Fig. 1, or also covers the submarine outcrops of the other two semi-permeable layers. Especially, the lower semi-permeable layer may also extend under the sea. These differences in the aquifer structure will not influence the mathematical model, as will be shown after the model equations below. The possibility of the existence of the submarine outlet-capping was discussed in Li et al. [9].
5. The density difference between the groundwater and the seawater can be neglected owing to its slight impact on groundwater level fluctuation [5].

Let the $x$-axis be positive landward and perpendicular to coastline, and the $z$-axis be vertical, positive upward (Fig. 1). The origin is located at the intersection of the coastline and the bottom of the lower semi-permeable layer. Using the assumptions above, the governing equations of the groundwater flow in the aquifer system can be written as

1. For the upper semi-permeable layer,

$$S_{s1}\frac{\partial h_1}{\partial t} = K_1\frac{\partial^2 h_1}{\partial z^2}, \quad -\infty < t < \infty, \quad M + M_2 < z < M_1 + M + M_2, \tag{1}$$

$$\left.\frac{\partial h_1}{\partial z}\right|_{z=M_1+M+M_2} = 0, \tag{2}$$

$$h_1(z,t;x)|_{z=M+M_2} = h(t,x), \tag{3}$$

where $h_1(z,t;x)$ denotes the hydraulic head [L] in the upper semi-permeable layer at the location $(x,z)$ and time $t$; $S_{s1}$ and $K_1$ represent the specific storativity [$L^{-1}$] and vertical hydraulic conductivity [$LT^{-1}$] of the upper semi-permeable layer, respectively; $M$, $M_1$ and $M_2$ are the thicknesses [L] of the confined aquifer, the upper and lower semi-permeable layers, respectively; $h(x,t)$ is the hydraulic head [L] of the confined aquifer at the instant $t$ and location $x$.