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Multi-resolution adaptive modeling of groundwater flow and transport problems

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Abstract

Many groundwater flow and transport problems, especially those with sharp fronts, narrow transition zones, layers and fingers, require extensive computational resources. In this paper, we present a novel multi-resolution adaptive *Fup* approach to solve the above mentioned problems. Our numerical procedure is the Adaptive Fup Collocation Method (AFCM), based on *Fup* basis functions and designed through a method of lines (MOL). *Fup* basis functions are localized and infinitely differentiable functions with compact support and are related to more standard choices such as splines or wavelets. This method enables the adaptive multi-resolution approach to solve problems with different spatial and temporal scales with a desired level of accuracy using the entire family of *Fup* basis functions. In addition, the utilized collocation algorithm enables the mesh free approach with consistent velocity approximation and flux continuity due to properties of the *Fup* basis functions. The introduced numerical procedure was tested and verified by a few characteristic groundwater flow and transport problems, the Buckley–Leverett multiphase flow problem, the 1-D vertical density driven problem and the standard 2-D seawater intrusion benchmark–Henry problem. The results demonstrate that the method is robust and efficient particularly when describing sharp fronts and narrow transition zones changing in space and time.

Keywords: *Fup* basis functions; Compact support; Method of lines; Adaptive *Fup* collocation method; Multi-resolution approach; Numerical dispersion; Groundwater flow and transport problems

1. Introduction

Many groundwater flow and transport problems exhibit a wide range of space and/or temporal scales characterized by sharp gradients resulting in fingering and layering with the existence of sharp interface and narrow transition zones. These characteristics are commonly present in problems of unsaturated and multiphase flow [5,13,23,34], density driven flow and transport [6,17,43] and reactive transport [3,39].

The numerical modeling of such processes usually presents significant difficulties in resolving numerical oscillations and dispersion. In order to overcome these difficulties demanding computational resources with a very fine grid and small time steps are needed. In recent numerical approaches adaptive methods with low computational costs are being developed. The first attempt to apply them has been focused upon using classical finite difference and finite element methods [2,17]. The main difficulty in applying these methods is finding a stable solution at the transition between zones having different discretization. Significant improvements have been obtained by the adaptive discontinuous finite element method, e.g. [5,34]. Recently, there have been many attempts to develop new adaptive procedures which, among others, are focused upon using the adaptive wavelet Galerkin methods [9,12] and collocation methods [7,8,15,25,26,46,47]. The adaptive wavelet Galerkin methods have three potential difficulties: treatment of general boundary conditions, treatment of

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nonlinearities and solving problems with complex domains. The first two difficulties can be successfully solved using the collocation procedure, while the third is still an open research topic.

The wavelet's main feature is to facilitate the grid space adaptation and multi-resolution data compression. It enables solving problems with a sharp interface and narrow transition zone by changing their location and steepness in time and space. Wavelets utilize non-uniform grids dynamically adaptive according to the solution development. Any function, signal or data can be represented by a linear combination of basis functions (discrete wavelet transform) in multi-resolution fashion with different scales or frequencies and locations. This means that specific frequencies are associated with a particular spatial location that is not possible in classical Fourier transform (coefficients of linear combinations are wavelet coefficients which are associated with a specific resolution level [scale or frequency] and [collocation] point in space/time domain). This procedure is also known as multi-resolution analysis (MRA). Besides the solution variables (pressure, concentration, velocity) any other variables can be represented in the same multi-resolution fashion such as electrical or hydraulic conductivity, porosity, natural recharge or pumping. Furthermore, it is possible to use other basis functions with compact support (non-zero values only in one part of the domain) within the collocation method [46]. The spline adaptive collocation methods are described in [10,36,49].

Apart from wavelets and splines, there is a relatively lesser known class of atomic or R_{bf} basis functions (Rvachev's basis functions) [37,38]. Atomic functions are classified between classic polynomials and spline functions. However, in practice, their application as basis functions is closer to splines or wavelets. In this paper we use *Fup* basis functions which are one type of atomic basis functions. Gotovac and Kozulić [20] systemized the existing knowledge on atomic functions and presented its usage and calculation into a numerically applicable form. The application of *Fup* basis functions has been demonstrated in signal processing [32,51], in initial value problems [21], and in the non-adaptive collocation method for boundary value problems [22,31].

The main objective of this paper is to develop the **A**daptive **F**up **C**ollocation **M**ethod (AFCM) and demonstrate its application to groundwater flow and transport problems. Presented is a novel adaptive *Fup* collocation method which is well suited to dealing with strong nonlinear groundwater problems with sharp fronts and narrow transition zones. A numerical procedure is implemented through a method of lines. Spatial discretization and grid adaptation are obtained by *Fup* collocation transform, while time integration is obtained by solving the system of **D**ifferential-**A**lgebraic **E**quations (DAE). Furthermore, this method enables the adaptive multi-resolution evolution of a solution with resolved spatial and temporal scales and a desired level of accuracy. The numerical method has been tested and verified with a few characteristic groundwater flow and transport examples.

The following section presents a brief review of *Fup* basis functions. Section 3 describes the proposed collocation method; numerical examples are in Section 4, followed by conclusions in Section 5. Finally, two appendixes are added in order to show construction of the finite difference operator and convergence properties of the AFCM.

2. *Fup* basis functions preliminaries

Atomic or Rvachev's basis functions – R_{bf} have a compact support and they are infinitely differentiable functions [20,37]. They are classified in between classical polynomials and spline functions, but in practice their application as basis functions is closer to splines and wavelets.

Atomic functions, $y(\cdot)$, are defined as solutions of differential-functional equations of the following type:

$$Ly(x) = \lambda \sum_{k=1}^{M} C_k y(ax - b_k) \tag{1}$$

where L is a linear differential operator with constant coefficients, λ is a scalar different than zero, C_k are coefficients of the linear combination, $a > 1$ is a parameter defining the length of the compact support and b_k are coefficients which determine displacements of basis functions.

The simplest function, which is the most studied among atomic basis functions, is the $up(x)$ function (Fig. 1a). Function $up(x)$ is a smooth function with compact support $[-1, 1]$, which is obtained as a solution of a differential-functional equation

$$up'(x) = 2up(2x + 1) - 2up(2x - 1) \tag{2}$$

with the normalized condition

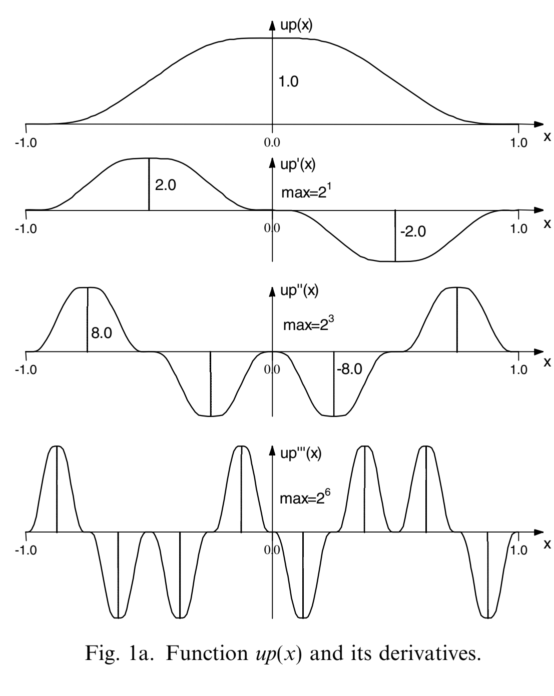

Fig. 1a. Function $up(x)$ and its derivatives.