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A two-phase grey fuzzy optimization approach for water quality management of a river system

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Abstract

An extension of the Grey Fuzzy Waste Load Allocation Model (GFWLAM) developed in an earlier work is presented here to address the problem of multiple solutions. Formulation of GFWLAM is based on the approach for solving fuzzy multiple objective optimization problems with max–min as the operator, which usually may not result in a unique solution. The multiple solutions of fuzzy multiobjective optimization model should be obtained as parametric equations or equations that represent a subspace. A two-phase optimization technique, two-phase GFWLAM, is developed to capture all alternative or multiple solutions of GFWLAM. The optimization model in Phase 1 is exactly same as the optimization model described in GFWLAM. The optimization model in Phase 2 maximizes the upper bounds of fractional removal levels of pollutants and minimizes the lower bounds of fractional removal levels of pollutants keeping the value of goal fulfillment level same as obtained from Phase 1. The widths of the interval-valued fractional removal levels play an important role in decision-making as these can be adjusted within their intervals by the decision-maker considering technical and economic feasibility in the final decision scheme. Two-phase GFWLAM widens the widths of interval-valued removal levels of pollutants, thus enhancing the flexibility in decision-making. The methodology is demonstrated with a case study of the Tunga-Bhadra river system in India.

Keywords: Fuzzy optimization; Grey optimization; Membership functions; Uncertainty; Water quality management

1. Introduction

Waste Load Allocation (WLA) or water quality management in a stream refers to the determination of required pollutant [e.g., Biochemical Oxygen Demand (BOD) loading, toxic pollutant concentration, etc.] treatment levels at a set of point sources of pollution to ensure that water quality is maintained at desired levels throughout the stream. A WLA model for decision-making in water quality control in a river system, in general, integrates a water quality transport or simulation model with an optimization model to provide best compromise solutions acceptable to both Pollution Control Agency (PCA) and dischargers (e.g., municipal and industrial dischargers). Almost four decades of developments in uncertainty modeling for WLA have opened up new avenues for incorporating uncertainties in models [29]. In a typical WLA problem uncertainty due to randomness arises mainly due to random nature of input parameters, viz., streamflow, effluent flow, rate coefficients, temperature, etc; and that due to imprecision or fuzziness is associated with setting up water quality standards and goals of PCA, dischargers. The probabilistic approach and fuzzy approach are the concepts to address these two kinds of uncertainties. The first one addresses uncertainty due to randomness, arising primarily because of the random variations of model parameters, and second one addresses uncertainty due to imprecision or fuzziness, arising from uncertain quantification of management goals. Uncertainty due to randomness has been

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addressed extensively in the WLA models of river systems, starting with the pioneering work of Loucks and Lynn [25]. Uncertainty due to imprecision in setting up standards, and defining management goals of the stakeholders in WLA can be modeled by fuzzy set theory [39]. Thus, probabilistic approach and fuzzy logic approach serve two different purposes on modeling uncertainty in a waste load allocation problem.

Jowitt and Lumbers [15] first used fuzzy sets to describe the water quality standards through linguistic description of water quality. Hathhorn and Tung [12] addressed the waste load allocation problem in a fuzzy optimization [40,41] framework. The concept of fuzzy sets is extensively used in water quality management problems in many earlier works [4,10,18–21,26–28,33,36,37]. A major limitation in all such models is that the boundaries [32] of membership functions (also called the membership parameters) are assumed fixed and values are assigned to the parameters based on experience and judgment. As the model results are likely to vary considerably with change in the membership functions [4,26], uncertainty in the membership parameters and shape of the membership functions should be addressed in fuzzy optimization models for water quality management. In practical situations different water quality standards for surface water are used for different uses for a water quality indicator. For example, standards for public water supply, industrial water supply, agricultural water supply, fish propagation and wild life may all be different for the same water quality indicator, DO [11]. This results in an uncertainty in the membership parameters and leads to a second level of fuzziness in the model, with the membership functions themselves being imprecisely stated. Karmakar and Mujumdar [16] developed the Grey Fuzzy Waste Load Allocation Model (GFWLAM) for relaxing the values of membership parameters by treating them as interval grey numbers [13,14,24]. An interval grey number ($x^{\pm}$) is a closed and bounded interval with known lower (x^{-}) and upper (x^{+}) bounds but unknown distribution information [14,24]

$$x^{\pm} = [x^{-}, x^{+}] = [t \in x | x^{-} \leqslant t \leqslant x^{+}] \tag{1}$$

$x^{\pm}$ results in a 'deterministic number' or 'white number' when, $x^{\pm} = x^{-} = x^{+}$. A terminology 'imprecise membership function' is used to represent the membership functions with uncertain parameters. A previously developed Fuzzy Waste Load Allocation Model (FWLAM) [34] for a river system forms the basis of GFWLAM that addresses uncertainty in the membership functions for the fuzzy goals of the PCA and the dischargers using the concept of grey systems. The imprecision in specifying the water quality criteria and fractional removal levels of pollutant ($\hat{X}$) was modeled in a fuzzy framework.

The GFWLAM [16] is a more flexible form of FWLAM [34] developed earlier, as it provides the solutions as interval grey numbers, rather than a solution as deterministic or white number. Formulation of GFWLAM is based on the formulation of fuzzy multiple objective optimization problem proposed by Zimmermann [41] using max–min as the operator, which usually may not result in a unique solution [7,17,23]. Multiple solutions of fuzzy multiple objective optimization model should be obtained as a parametric equation or equation that represents a subspace. Determination of such a subspace in a fuzzy multiple objective optimization problem is itself a potential area for research [17,22,23]. In the present work a two-phase optimization model is developed to capture all the alternative or multiple solutions of GFWLAM, providing more option to the decision-maker by widening the range of optimal solutions. Zimmermann [41] first used the max–min operator of Bellman and Zadeh [2] to solve a fuzzy multiple objective linear programming problem, stated as follows:

$$\text{Maximize} \quad \lambda \tag{2}$$

$$\text{subject to} \quad f_i((ax)_i) \geqslant \lambda; \quad \forall i = 1, 2, \ldots, m \tag{3}$$

$$Bx \leqslant b \tag{4}$$

$$x \geqslant 0 \tag{5}$$

where x is a n-dimensional vector with elements $x_1, x_2, \ldots, x_n$; $Bx \leqslant b$ are system constraints in vector notation; and $f_i((ax)_i)$ is the membership function for ith fuzzy goal, a is coefficient of x, and λ is the minimum goal fulfillment level. In such a max–min formulation of a fuzzy optimization model, different sets of optimal values of x (i.e., $\hat{x}$) may exist corresponding to the unique optimal value of λ (i.e., $\hat{\lambda}$), resulting in multiple solutions. Existence of multiple solutions in the max–min formulation of fuzzy optimization model is very common. In any Linear Programming (LP) problem, whenever there are two vertices which are optimal, we can always generate all other optimal solutions by the 'convex linear combination' of coordinates of the two solutions, which will capture all the decision alternatives. In a similar sense, a methodology is necessary to capture all the decision alternatives from any fuzzy multiple objective optimization model. It is difficult to pinpoint the exact modeling features that contribute to a possibility of multiple solutions in a fuzzy optimization model. It is, however, observed that as the number of objectives and decision variables increases in the fuzzy multiple objective optimization model, the possibility of multiple solutions increases. In a fuzzy decision-making problem, all solutions from the fuzzy multiple objective optimization model should be obtained to determine the decision alternatives to facilitate the decision-making. A large case study of Tunga-Bhadra river system in India is furnished in Karmakar and Mujumdar [16], for demonstrating the application of GFWLAM, introducing a number of objectives (viz., goals of the PCA at 30 checkpoints, where the concentrations of water quality indicators are measured, and goals of the eight dischargers) and decision variables (viz., fractional removal levels of eight dischargers) in the optimization model, which results in an increase in possibility of obtaining multiple solutions. The Tunga-Bhadra river system has 8 dischargers, but all of them are not equally critical and sensitive to the interval-valued goal fulfillment