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Rainfall stochastic disaggregation models: Calibration and validation of a multiplicative cascade model

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Abstract

The simulation of long time series of rainfall rates at short time steps remains an important issue for various applications in hydrology. Among the various types of simulation models, random multiplicative cascade models (RMC models) appear as an appealing solution which displays the advantages to be parameter parsimonious and linked to the multifractal theory. This paper deals with the calibration and validation of RMC models. More precisely, it discusses the limits of the scaling exponent function method often used to calibrate RMC models, and presents an hydrological validation of calibrated RMC models. A 8-year time series of 1-min rainfall rates is used for the calibration and the validation of the tested models. The paper is organized in three parts. In the first part, the scaling invariance properties of the studied rainfall series is shown using various methods (q-moments, PDMS, autocovariance structure) and a RMC model is calibrated on the basis of the rainfall data scaling exponent function. A detailed analysis of the obtained results reveals that the shape of the scaling exponent function, and hence the values of the calibrated parameters of the RMC model, are highly sensitive to sampling fluctuation and may also be biased. In the second part, the origin of the sensivity to sampling fluctuation and of the bias is studied in detail and a modified Jackknife estimator is tested to reduce the bias. Finally, two hydrological applications are proposed to validate two candidate RMC models: a canonical model based on a log-Poisson random generator, and a basic micro-canonical model based on a uniform random generator. It is tested in this third part if the models reproduce faithfully the statistical distribution of rainfall characteristics on which they have not been calibrated. The results obtained for two validation tests are relatively satisfactory but also show that the temporal structure of the measured rainfall time series at small time steps is not well reproduced by the two selected simple random cascade models.

Keywords: Rainfall; Model; Stochastic; Disaggregation; Multifractal

1. Introduction

Long series of meteorological data, especially rainfall data, available at time and space scales suitable for hydrological uses [1,2] are not yet frequent. Most of the measured precipitation data are available at a daily time step. The networks of automatic gauges delivering data at shorter time steps are sparse (1 gauge for 500 km^2 in average in France), except in some dense urban areas. But even there, the rain gauge networks are relatively recent and the available short time step rainfall intensity series do generally not exceed 10–20 years. For various hydrological applications – i.e. flood frequency studies [3–5], urban drainage design [6], flood forecasting [7–9] – their is a need either for generating long series of probable rainfall scenarios or for a space-time disaggregation (i.e. downscaling) of available rainfall data.

A large variety of stochastic models or approaches have been proposed to achieve the simulation of rainfall. The

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direct approach, mainly used for point-rainfall process simulation initiated by Croley [10] describes a hyetograph as a series of juxtaposed elements: dry and rainy periods, and possibly showers and inter-showers within a rainy period. Each shower is described by its duration, rainfall amount, shape, peak position and intensity. The variables describing showers, inter-showers and dry period shapes constitute as many random variables whose probability density function (hereafter denoted PDF) can be fitted on measured data series. This approach has been used to simulate realistic rainfall scenarios for flow forecasting [7] and estimation of flood quantiles [11,4]. Some 5 or 10-min point rainfall simulators based on the direct approach and suited for urban hydrology [12,6] or for rural erosion studies [1] have been developed. The interesting results obtained with such models are nevertheless counterbalanced by the large amount of variables needed to describe the hyetograph shapes which comes against their real-world applicability.

The Poisson-cluster models, also mainly used for point-rainfall simulations and inspired by Neymann–Scott or Bartlett–Lewis models [13] are another popular approach. The occurrence time of the rain events are controlled by a Poisson process. Each rain event aggregates a number of elementary rectangular pulses (raincells). The cells duration and their intensity have an exponential PDF, possibly depending on the number of cells. Many research works have been performed to improve these models: modification in the model formulation, or changes in the variables PDF [14–18], influence of rain type on the model parameters [19], test of calibration procedures, evaluation on very long data series [20]. Poisson-cluster models require less parameters than models based on the direct approach, but to our knowledge, they have mainly been validated on their capability to reproduce rainfall quantiles at time steps exceeding 1 h [21,20,18]. These time steps are too large for applications in urban hydrology [2]. Other simulation methods have also been tested: chaotic models [22–24], artificial neural networks [25] as well as models based on the principle of simulated annealing [26]. But, among the possible modelling approaches, the multiplicative cascade disaggregation models, firstly introduced by Yaglom [27], appeared as promising and have therefore received a growing attention during the last decade [28–31]. Under certain conditions, these models produce rainfall series exhibiting scaling (multifractal) properties. This is in accordance with the results of the numerous multifractal analysis of rainfall series conducted since the founding works of Mandelbrot [32,33] on the fractal theory: rainfall data appear to show scaling invariance over a large range of space [34–38] and time steps [39–43]. The scaling nature of rainfall is still discussed [44,45]. Nevertheless, multiplicative cascade models appear as appealing rainfall simulation tools because of their link with the multifractal theory. They are moreover equally adapted for the simulation of rainfall in space and time, are parameter parsimonious, and are theoretically easy to calibrate and use as will be illustrated in the first part of this paper. Nevertheless, their calibration and validation give rise to some difficulties and questions which have seldom been commented on in previous papers. Moreover, results of stochastic rainfall model validations, have seldom been presented in previous publications [29,46].

The main objective of this study is to illustrate those questions on a given example: the calibration and validation of a random cascade rainfall model for the simulation of point-rainfall time series to be used in urban drainage design studies. The cascade models are examined here from the hydrologist's or model user's point of view, i.e. the main focus will be put on the practicality of the models in a 'real world' application. This paper is organised as follows. The first part presents the general principles of random multiplicative cascade models, the available rainfall data, their multifractal properties and the choice of a random cascade model. The robustness of the calibration of the model parameters (sensitivity to sampling fluctuation, possible biases) is discussed in the second part of the paper. The third and last part addresses the question of the validation of the selected cascade model for urban drainage design purposes. Its ability to reproduce correctly rainfall characteristics on which it has not been directly calibrated (intensity–duration–frequency curves, rainfall volumes over a given intensity threshold) is tested.

2. Part 1: Calibration of a random multiplicative cascade model

2.1. Random multiplicative cascades and their properties

Consider I_0 (resp. $R_0 = I_0 T$), the rainfall intensity (resp. amount) measured over a duration T. Fig. 1 represents the disaggregation of this quantity I_0 through a random cascade model. At each level of the cascade, I_0 is distributed over b sub-time steps ($b = 2$ in Fig. 1). The resulting intensities at a given level k of the cascade are equal to the prod-

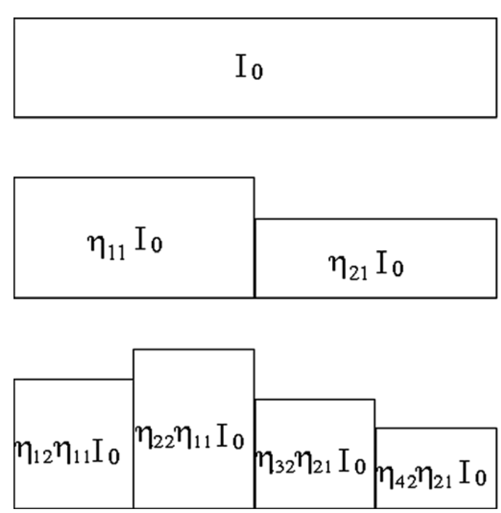

Fig. 1. Graphical representation of a two branches random cascade process.