Available online at www.sciencedirect.com

SCIENCE DIRECT®

Advances in Water Resources 29 (2006) 1088–1105

Advances in Water Resources

www.elsevier.com/locate/advwatres

Grey fuzzy optimization model for water quality management of a river system

Subhankar Karmakar, P.P. Mujumdar *

Department of Civil Engineering, Indian Institute of Science, Bangalore 560 012, India

Abstract

A grey fuzzy optimization model is developed for water quality management of river system to address uncertainty involved in fixing the membership functions for different goals of Pollution Control Agency (PCA) and dischargers. The present model, Grey Fuzzy Waste Load Allocation Model (GFWLAM), has the capability to incorporate the conflicting goals of PCA and dischargers in a deterministic framework. The imprecision associated with specifying the water quality criteria and fractional removal levels are modeled in a fuzzy mathematical framework. To address the imprecision in fixing the lower and upper bounds of membership functions, the membership functions themselves are treated as fuzzy in the model and the membership parameters are expressed as interval grey numbers, a closed and bounded interval with known lower and upper bounds but unknown distribution information. The model provides flexibility for PCA and dischargers to specify their aspirations independently, as the membership parameters for different membership functions, specified for different imprecise goals are interval grey numbers in place of a deterministic real number. In the final solution optimal fractional removal levels of the pollutants are obtained in the form of interval grey numbers. This enhances the flexibility and applicability in decision-making, as the decision-maker gets a range of optimal solutions for fixing the final decision scheme considering technical and economic feasibility of the pollutant treatment levels. Application of the GFWLAM is illustrated with case study of the Tunga–Bhadra river system in India.

Keywords: Decision-making; Fuzzy interval analysis; Fuzzy mathematical programming; Grey systems theory; Interval analysis; Optimization; Uncertainty; Water quality management

1. Introduction

Decision-making for water quality management in a river system is addressed by Waste Load Allocation (WLA) models. A WLA model, in general, integrates a water quality simulation model with an optimization model to provide best compromise solutions acceptable to both PCA and dischargers. A number of WLA models have been developed in the past for optimal allocation of assimilative capacity of a river system [3,10,25]. Uncertainty due to randomness has been addressed extensively in the models for water quality management of river systems, starting with the pioneering work of Loucks and Lynn [25] (i.e., [3,9,10]). Another type of uncertainty in water resources problems is the imprecision in goals and model parameters in general has been addressed with fuzzy sets [2,39]. The concept of fuzzy decision is also used in water quality management problems in some recent work [4,29,30,33,34,37,38]. Sasikumar and Mujumdar [34] developed a Fuzzy Waste Load Allocation Model (FWLAM) for water quality management of a river system. A useful feature of FWLAM is its capability to incorporate the conflicting goals of PCA and dischargers in a fuzzy optimization framework. Imprecision associated with the goals of

* Corresponding author. Tel.: +91 80 2360 0290/2293 2669; fax: +91 80 2360 0290.
E-mail address: pradeep@civil.iisc.ernet.in (P.P. Mujumdar).
URL: http://civil.iisc.ernet.in/~pradeep (P.P. Mujumdar).

0309-1708/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.advwatres.2006.04.003

PCA and dischargers are modeled in a fuzzy environment. Sasikumar and Mujumdar [33] extended this work to include uncertainty due to randomness and imprecision simultaneously. Two levels of uncertainty, one associated with "low water quality" and other with "low risk" are quantified and incorporated in the management model using the concept of fuzzy probability. Considering seasonal variation in streamflow for specifying variable fractional removal policy, Mujumdar and Sasikumar [29] developed a procedure to evaluate the fuzzy risk of low water quality in a season.

A major limitation in the models of Sasikumar and Mujumdar [33,34] and Mujumdar and Sasikumar [29] is that the lower and upper bounds of the membership functions (membership parameters) are assumed fixed and values are assigned based on some judgement (such as, for example, a lower bound of 5 mg/L and an upper bound of 9 mg/L for DO concentration in a river system). As the model results are likely to vary considerably with change in the membership functions [4,29], uncertainty in the bounds and shape of the membership functions should be addressed in fuzzy optimization models for water quality management. The present work is aimed at relaxing the lower and upper bounds of the membership functions by treating them as interval grey numbers [24,28], thus providing a range of "best compromise" solutions to impart more flexibility in water quality management decisions. The Grey Fuzzy Waste Load Allocation Model (GFWLAM) developed in the present work, is a more flexible form of FWLAM developed earlier. In FWLAM the imprecision associated with management goals are quantified using membership functions. These membership functions are subjective and depend on the particular problem being solved. To address uncertainty in the lower and upper bounds of membership functions, the membership functions themselves are treated as fuzzy in GFWLAM and the membership parameters are expressed as "interval grey numbers". This is achieved by using the grey fuzzy linear programming technique [12,13,42]. The model uses the concepts of inexact programming [6,16,17,20,26,40], grey programming [4,5,12–15,18,24], and fuzzy multiobjective optimization technique [29,32–34,41]. The uncertainty in the parameters of membership functions in fuzzy mathematics can be modeled also by using the concepts of interval programming [28,35] and type-2 membership functions [27]. A concept of "imprecise fuzzy decision" is introduced to provide a range of acceptable decisions. Optimal values of the objective function and decision variables are obtained in the form of interval grey numbers, enhancing flexibility in decision-making.

The next section "Grey Fuzzy Waste Load Allocation Model (GFWLAM)", gives a description on GFWLAM. It also provides an overview of basic concepts of grey systems. Description of grey fuzzy goals for water quality management of a general river system, concept of imprecise fuzzy decision, and description of mathematical model formulation are given in the same section. The methodology is demonstrated through a case study of the Tunga–Bhadra river system in the "Model application" section. Results, discussion and conclusions are presented in subsequent sections.

2. Grey Fuzzy Waste Load Allocation Model (GFWLAM)

The FWLAM developed earlier for water quality management of a river system is formulated as a deterministic problem. It forms the basis for the optimization model developed in the present work. Following Sasikumar and Mujumdar [34], the model is described using a general river system. The system consists of a set of dischargers, who are allowed to release pollutants into the river after removing some fraction of the pollutants. Fractional removal (treatment) is necessary to maintain acceptable water quality condition in the river as prescribed by the PCA. Acceptable water quality condition is ensured by checking the water quality in terms of water quality indicator levels (e.g., DO-deficit level) at a finite number of locations, which are referred to as checkpoints. The goals of PCA are to ensure that pollution is within an acceptable limit by imposing water quality and effluent standards. On the other hand, the dischargers prefer to use the assimilative capacity of the river system to minimize the waste treatment cost, by assigning aspiration level (minimum desirable treatment) and maximum fractional removal level for different pollutants. These goals are imprecise and subjective, and are addressed in the model in a fuzzy mathematical framework by assigning membership functions. The concentration of the water quality indicator is expressed as a function of fractional removal levels of the pollutants. A fuzzy optimization problem is formulated with the set of fractional removal levels and degree of goal satisfaction as the decision variables.

Choice of appropriate bounds and shape of membership functions is an important issue in any fuzzy optimization model. In FWLAM the uncertainty in the lower and upper bounds of the membership functions was not considered. These bounds depend on the desirable and maximum permissible level of water quality indicators (e.g., DO-deficit, hardness, nitrate–nitrogen concentration). In practical situations different water quality standards for surface water are used for different uses for a water quality indicator. For example, standards for public water supply, industrial water supply, agricultural water supply, fish propagation and wild life may all be different for the same water quality indicator, DO [11]. It results in an uncertainty in the lower and upper bounds of the membership functions. This leads to a