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Acoustic backscatter inversion for suspended sediment concentration and size: A new approach using statistical inverse theory

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ABSTRACT

A new method is introduced for estimating suspended sediment concentration and grain size from acoustic backscatter observations, incorporating techniques from linearized statistical inverse theory and data assimilation. A series of laboratory experiments with a sediment-laden jet were conducted for the purposes of demonstrating the method. The inversion results show improvements in stability compared to existing methods, in cases with highly attenuating suspensions. These improvements are due to the use of statistical regularizing assumptions, which act to mitigate issues associated with poor conditioning and nonlinearity in the inverse problem. The new method is also more computationally efficient than existing methods, owing to the use of linearization. Matlab code is provided implementing the new method, see Appendix A.

1. Introduction

Acoustic backscatter inversion has been successfully used to characterize the concentration and grain size of sediments suspended in water, in laboratory and field studies (e.g., Hanes et al., 1988; Vincent et al., 1991; Hay and Sheng, 1992; Osborne et al., 1994; Hurther et al., 2011; Ruessink et al., 2011; O'Hara Murray et al., 2012; Aagaard, 2014; also see reviews by Thorne and Hanes, 2002; Thorne and Hurther, 2014; and references therein). The principle behind its use involves inversion of a well-established model which describes the average backscattered amplitude from a random field of suspended scatterers. By inverting this model, one can use acoustic backscatter observations to infer the size and concentration of the scatterers. Using this technique, acoustic instruments are capable of collecting sediment measurements at high spatial and temporal resolution, and at far enough range to be considered non-intrusive.

In practice, however, there are several technical barriers to the application of acoustic backscatter inversion. The acoustic scattering properties of natural sediments vary based on their shape and mineralogical composition (e.g., Schaafsma and Hay, 1997; Moate and Thorne, 2012), meaning detailed site-specific calibration is required to obtain unbiased results. The instrument itself must also be calibrated (Betteridge et al., 2008; Stanton and Chu, 2008). And other site characteristics such as water temperature, salinity, and sediment size distribution need to be known.

The present work is focused on another, more fundamental, barrier to accurate inversion: suspended sediment backscatter is inherently statistical, i.e. observations are corrupted by statistical uncertainty and/or noise. This complicates the inversion problem for two reasons. First, the acoustic backscatter model is nonlinear, such that observational noise can lead to large inversion errors when using least-squares-type inversion methods, because of the presence of multiple local minima. This problem is compounded by conditioning issues which occur when the suspension is highly attenuating—that is, when the backscatter at a given range bin depends strongly on the backscatter at previous range bins. In such cases, statistical noise and model errors accumulate along the profile, leading to "blowup" of the inversion. High attenuation occurs when sediment concentration is high, or when high acoustic frequencies are used; high statistical noise occurs with small measurement ensemble sizes. These factors are at odds with measuring at high spatial and temporal resolution in the bottom boundary layer, which is part of the motivation for the present work.

Given the issues just described, inverse algorithms generally need to be stabilized by adding extra constraints, in the form of measurements or assumptions. A promising approach is to incorporate direct measurements of total along-profile attenuation. Thorne et al. (1995) demonstrated that the bed echo can be used for this purpose, although this is not possible if the bed echo is signal-saturated (Ruessink et al., 2011). Similarly, Shen and Lemmin (1998) used a system with two facing transducers in order to

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directly measure and correct for attenuation, however this would seem impractical for non-intrusive profiling applications. Stability can also be improved in cases where the grain size vs. range is known *a priori* (Thorne et al., 2011).

The present work offers a new approach to the stabilization/regularization problem, wherein constraints are added based on statistical assumptions (Section 2). Specifically, the time-dependent sediment concentration and grain size are constrained using *a priori* assumptions regarding (a) the time-averaged state, (b) the magnitude of time-variations, and (c) the statistical variability of the measured data. This information is incorporated using techniques from linearized statistical optimization and data assimilation—for further background on such methods, see, e.g., Aster et al. (2013) for a statistical perspective, or Lewis et al. (2006) for a data assimilation perspective. The statistical method is demonstrated alongside two existing inversion methods, using laboratory observations with a turbulent particle-laden jet (Sections 3 and 4). The results show the statistical method to be stable, accurate, and efficient, even in highly attenuating conditions. An implementation of the method in Matlab code is provided in Appendix A.

2. Methods

2.1. Acoustic backscatter model

The theory for acoustic backscatter amplitude from a dilute suspension of particles has been previously described in several studies (e.g., Hay, 1983, 1991; Thorne et al., 1991). A recent review with a focus on inversion was given by Thorne and Hurther (2014); their formulation and notation will be outlined here, and is described in further detail by Thorne and Hardcastle (1997).

Consider a single transducer which insonifies a small volume of suspended particles at range r. The root-mean-square received (backscattered) amplitude, written here as V, can be shown to be

$$V = \frac{K_s K_t}{\psi r} M^{1/2} e^{-2\alpha r}, \tag{1}$$

where

$$K_s = \left(\frac{f_a^2}{\rho_s a}\right)^{1/2}, \quad K_t = \frac{0.96}{ka_t} V_0 r_0 \left[\frac{3c\tau}{16}\right]^{1/2}. \tag{2}$$

In these equations, a is the (effective) particle radius, ρ_s is the particle density, and M is the mass concentration (other variables are described later). In an observational system, acoustic range-gating is used to measure a profile of V in discrete range bins having width Δr. It is assumed that the suspension is uniform within each bin, such that the discretized form of the above equations is straightforward: each range bin r_i is associated with a discrete value of a_i and M_i, etc., and integrals are converted to sums.

System calibration factors, which are assumed known, are encapsulated in K_t/ψ. This includes the acoustic wavenumber and phase speed in water, k and c, the pulse duration τ, the effective transducer radius a_t, and a reference value $V_0 r_0$. The factor $0.96/ka_t$ is an approximation (Thorne and Hardcastle, 1997) accounting for integration over the transducer directivity pattern. The factor ψ accounts for near-field spreading effects, as given by Downing et al. (1995)

$$\psi = \frac{1 + 1.35z + (2.5z)^{3.2}}{1.35z + (2.5z)^{3.2}}, \tag{3}$$

where $z = 2r/ka_t^2$.

The intrinsic backscattering properties of the suspension are encapsulated by the terms χ and f_a, which are the normalized total scattering cross section and form factor, respectively, both of which are assumed to depend only on the normalized wavenumber $x = ka$. The form factor appears as part of the factor K_s. The normalized total scattering cross section affects along-path attenuation of the acoustic signal, via the attenuation coefficient $\alpha = \alpha_w + \alpha_s$, in which $\alpha_w(f)$ is attenuation due to water, and α_s is attenuation due to sediments

$$\alpha_s = \frac{1}{r}\int_0^r \frac{3\chi}{4a\rho_s} M \, dr. \tag{4}$$

For uniform elastic spherical scatterers, χ and f_a can be exactly specified (Faran, 1951). For natural sediments, variability in particle shape, composition, and size distribution generally necessitates empirical formulations for χ and f_a, which has been the subject of several laboratory studies (Sheng and Hay, 1988; Schaafsma and Hay, 1997; Thorne and Buckingham, 2004; Thorne and Meral, 2008; Moate and Thorne, 2012).

2.2. Direct inversion

Various methods (Thorne et al., 1991; Hay and Sheng, 1992; Crawford and Hay, 1993; Lee and Hanes, 1995; Thosteson and Hanes, 1998; Thorne et al., 2011; among others) have been proposed to invert Eq. (1), i.e. to determine particle size and concentration profiles based on observations of V at two or more frequencies. These methods seek to determine $M(r)$ and $a(r)$ such that the modeled backscatter (Eq. (1)) fits the observations as closely as possible, as defined by a specified criterion or cost function. The cost functions that have been considered in existing methods do not attempt to account for statistical errors/noise in the observations or the model; such methods will be referred to here as "direct inversion" methods. A review of such methods was recently given by Thorne and Hurther (2014), who noted that although several methods have been developed, they generally have similar performance.

Two direct inversion methods are considered here, for the purpose of comparison to the statistical method of Section 2.3. The first is the method of Thorne et al. (2011), which was implemented based on Matlab code provided by the authors of Thorne and Hurther (2014) (see Internet link in their article). This will be referred to as the "Thorne et al. method". In outline, this method proceeds as follows. For range bin i, a direct search of parameter space is conducted by solving Eq. (1) for M_i over a user-specified set of candidate values for a_i, separately for each of the observed acoustic frequencies. The contribution of sediment-induced attenuation from bin i itself is incorporated using an iterative scheme. An optimum value of a_i (and hence M_i) is then chosen from the candidate a_i, according to the minimum value of $\langle M - \langle M\rangle\rangle^2/\langle M\rangle$, where $\langle\rangle$ denotes averaging over the observed acoustic frequencies. The analysis then proceeds to bin $i+1$.

The second direct inversion method considered here is an adaptation of the "ratio method" originally proposed by Hay and Sheng (1992). For range bin i, the ratio method solves for a_i and then M_i using two separate inversion steps. The first step solves for a_i by considering ratios of backscatter amplitude at different frequencies. In the implementation used here, the value of a_i is chosen by minimizing errors in backscatter ratios over all possible pairs of frequencies, using the following cost function:

$$J_a = \sum_{n \neq m} \left[\log \frac{V(f_n)}{V(f_m)} - \log \frac{V^o(f_n)}{V^o(f_m)} \right]^2, \tag{5}$$

where $V^o(f_n)$ is the observed backscatter amplitude at frequency f_n, and $V(f_n)$ is the corresponding model prediction using Eq. (1).