Continental Shelf Research 30 (2010) 680–691

Contents lists available at ScienceDirect

Continental Shelf Research

journal homepage: www.elsevier.com/locate/csr

Intercomparison between finite element and finite volume approaches to model North Sea tides

Silvia Maßmann *, Alexey Androsov, Sergey Danilov

Alfred Wegener Institute for Polar and Marine Research, Postfach 12-01-61, 27515 Bremerhaven, Germany

ARTICLE INFO

Article history:
Received 11 November 2008
Received in revised form
24 June 2009
Accepted 3 July 2009

Topical issue on "Tides in Marginal Seas" (in memory of Professor Alexei V. Nekrasov). The issue is published with support of the North Pacific Marine Science Organization (PICES).
Available online 15 July 2009

Keywords:
North Sea
Finite element method
Finite volume method
Tidal models
Intercomparison

ABSTRACT

Unstructured meshes suggest a number of advantages in tide modeling by resolving coastlines and providing refinements where it is required. We investigate the performance of several unstructured grid methods (finite element and finite volume) and time stepping schemes with respect to their accuracy and computational cost in simulating the M_2 tide in the North Sea. On a triangular mesh, we compare solutions of one finite volume and two finite element approaches ($P_1-P_1, P_1^{NC}-P_1$) with the amplitude and phase of observation data. All models show reasonable agreement and we explain the differences. By comparing CPU times for one tidal cycle we get the computational efficiency of the temporal discretization schemes (Euler semi-implicit, leapfrog explicit, Runge–Kutta and Adams–Bashforth). Although numerical solvers involve more computational loads per time step, we give the preference to the semi-implicit models, as the increased time step size reduces the total computational time considerably.

1. Introduction

Tidal circulation in the ocean is sensitive to the geometry and bottom topography of the ocean basin. Unstructured grid tidal models, which can readily represent the complex shoreline and control the mesh density following the varying topography, suggest a number of advantages compared to structured finite difference (FD) models, especially in coastal regions. Over the last several decades, a number of models working on unstructured grids was proposed, as a rule, formulated for 2D shallow water equations.

One of the most popular approaches is based on the finite element (FE) method and uses the so-called generalized wave continuity equation (GWCE). It works on general triangular grids and represents the velocity and elevation as linear interpolations on elements. The approach is successfully exploited in models such as QUODDY (Lynch et al., 1996), ADCIRC (Westerink et al., 1992) and MOG2D (Carrére and Lyard, 2003), with a long record of applications ranging from local wind surge to global tide predictions. The other widely used approach utilizes a mixture of finite volume (FV) and finite difference methods and can be conceived of as a triangular analogue of C-grid discretization of quadrilateral grids (Casulli and Walters, 2000). Although they are widely used they are not free of shortcomings. The GWCE approach does not exactly satisfy the continuity equation as it is replaced by the GWCE to suppress elevation modes allowed by the arrangement of variables (velocity and elevation are at nodes). This leads to the loss of local mass conservation (see, e.g., Massey and Blain, 2006). The C-grid approach requires orthogonal meshes which makes mesh design less straightforward.

Recently, new promising methods were suggested as represented by $P_1^{NC}-P_1$ discretization for the FE approach (Hanert et al., 2004, 2005) and the discretization exploited by FVCOM (Chen et al., 2003) for the FV approach. They do not share the shortcomings of the methods mentioned above but maintain similar numerical efficiency. An FE analogue to C-grid method, utilizing RT_0 element, is gaining in popularity too. Its spurious velocity modes are naturally filtered out by viscosity (Hanert et al., 2003). Furthermore, even with mass matrix lumping, its dispersive properties are well preserved (Le Roux et al., 2009). On irregular meshes mass lumping has some implication on accuracy or mesh characteristics (see Walters et al., 2009 for a brief review and an intercomparison study) and should therefore be treated with caution.

* Corresponding author. Tel.: +49 471 4831 1878.
E-mail addresses:
Silvia.Massmann@awi.de (S. Maßmann), Alexey.Androsov@awi.de (A. Androsov), Sergey.Danilov@awi.de (S. Danilov).

0278-4343/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.csr.2009.07.004

Many other approaches have been discussed in the literature and used to formulate models. They are less known to us and are not mentioned here. In particular, on the side of FE method, there is growing interest to discontinuous Galerkin methods which provide higher numerical accuracy but also impose a larger computational burden as the number of degrees of freedom per element also increases.

Given the multitude of existing unstructured-grid models a question emerges on their relative accuracy and numerical efficiency. This paper suggests a step aimed at partly answering this question by proposing an intercomparison study of several recent methods with some modifications introduced to their time stepping algorithm. As an object of numerical study we have selected tides in the North Sea. The North Sea is a well-explored domain (see, e.g., Defant, 1961; Huess, 2000; Klein et al., 1994; Toro et al., 2005) characterized by complex morphometric features and very high tidal activity. The tidal observations are reliable and numerous which makes this area well suited for model validation. On the other hand, it is of a limited size so that overall computational burden remains modest.

For the intercomparison we use seven models exemplifying finite volume (FV) and finite element (FE) classes. They are run with identical settings on a North Sea and Baltic Sea triangular mesh. The FV approach uses spatial discretization of FVCOM (Chen et al., 2003) with fourth order Runge–Kutta (FVRK), third order Adams–Bashforth (FVAB) and semi-implicit time stepping (FVSI). Except for sharing discretization, our implementation has no other links to FVCOM.

The FE class is represented by two methods. The first one follows Hua and Thomasset (1984) and uses continuous P_1 elements for the elevation and non-conforming discontinuous P_1^{NC} elements for the velocity with a leapfrog explicit (NCLF), Adam–Bashforth (NCAB) and Euler semi-implicit time stepping (NCSI). In the second FE approach, the velocity and elevation are both represented by P_1 functions (P1P1). Here we implemented only the semi-implicit time stepping and spurious elevation modes are suppressed by a particular form of the pressure correction method employed.

Our selection misses FE models based on GWCE as they are relatively well documented, and triangular C-grid model as they require orthogonal meshes. The RT_0 case with consistent mass matrix was not included too and presents an obvious choice for subsequent work.

The absence of simple rules to find neighborhood information on unstructured meshes requires using look-up tables which increases the computational time. For the same number of degrees of freedom the computational cost of FE and FV methods is larger than that of methods formulated on structured meshes. To fully exploit advantages of geometric flexibility suggested by unstructured meshes the models still need efficient numerical algorithms to be competitive to or outperforming the FD codes. Within the unstructured-grid class of models, the comparison of real computational cost and accuracy between FV and FE models is of interest as it is commonly expected that FV schemes are more efficient computationally but less accurate. In the literature, there are only a few studies comparing FV and FE discretizations of the shallow water equations (see, e.g., Lukáčová-Medvid'ová and Teschke, 2006; Walters et al., 2009), so we hope that our study will be a helpful contribution.

The paper is organized as follows. In Section 2 the description of models with their spatial and time discretization is presented. Section 3 contains results of modeling the dominant tidal wave M_2 in the North Sea using these models. The results are compared among themselves and against available observational data and the computational efficiencies of algorithms selected for this study is analyzed. In Section 4 conclusions of the work are formulated.

2. Model description

2.1. Shallow water equations

The equations under consideration are the 2D shallow water equations. As known from literature (see, e.g., Pedlosky, 1987; Anderson, 1995) they are derived from vertically integrating the Reynolds-averaged Navier–Stokes equations under the hydrostatic assumption and Boussinesq approximations. The momentum equation is taken in non-conserving form for the FE codes (NCLF, NCAB, NCSI, P1P1)

$$\partial_t \mathbf{u} + f\mathbf{k}\times\mathbf{u} + g\nabla\eta + (\mathbf{u}\cdot\nabla)\mathbf{u} = H^{-1}\nabla\cdot(AH\nabla\mathbf{u}) - rH^{-1}|\mathbf{u}|\mathbf{u} \quad (1)$$

and in the conserving, flux form

$$\partial_t \mathbf{U} + Hf\mathbf{k}\times\mathbf{u} + gH\nabla\eta + \nabla\cdot(\mathbf{U}\mathbf{u}) = \nabla\cdot(AH\nabla\mathbf{u}) - r|\mathbf{u}|\mathbf{u} \quad (2)$$

for the FV codes (FVAB, FVRK and FVSI). Here $\mathbf{u}$ is the horizontal velocity, $\mathbf{U} = H\mathbf{u}$ is the transport and $H = \eta + H_0$ the total fluid depth with H_0 the unperturbed water depth and η the deviation thereof. f is the Coriolis parameter, $\mathbf{k}$ the upward unit vector, r the bottom friction coefficient, g the gravitational acceleration and A the eddy viscosity coefficient. Noteworthy, the form of viscosity operator is different in both cases which is done for numerical convenience.

The continuity equation takes the form

$$\partial_t \eta + \nabla\cdot\mathbf{U} = 0 \quad (3)$$

for the FV codes or equivalently

$$\partial_t \eta + \nabla\cdot(\eta + H_0)\mathbf{u} = 0 \quad (4)$$

for the FE codes (NCLF, NCAB, NCSI, P1P1).

2.2. Boundary and initial conditions

The set of equations derived above is related to the type of incompletely parabolic equations (see Gustafsson and Sundström, 1978). We denote $\partial\Omega$ the boundary split into the solid part, $\partial\Omega_1$, and the open part, $\partial\Omega_2$, where we have

$$\mathbf{u}_n|_{\partial\Omega_1} = 0, \quad \Gamma(\mathbf{u},\eta)|_{\partial\Omega_2} = \mathbf{\Psi}, \quad (5)$$

where $\mathbf{u}_n$ is the velocity normal to $\partial\Omega_1$, Γ is the operator of the boundary conditions and $\mathbf{\Psi}$ is the known vector-function determined by the boundary regime as discussed in Oliger and Sundström (1978). In practice, the necessary information on the open boundary is unavailable, and the common choice is, in place of the second condition (5), either to prescribe level oscillations $\eta|_{\partial\Omega_2}$ or impose radiation boundary condition $\mathbf{u}_n = \mathbf{u}\cdot\mathbf{n} = \sqrt{(g/H_0)}\eta$ which would provide free passage of linear waves (in the absence of Coriolis effect). Here $\mathbf{n}$ is the outer unit normal to $\partial\Omega_2$. Accuracy of the reduced boundary-value formulation when only the sea level is assigned at the open boundary regardless of the boundary regime was considered by Androsov et al. (1995).

Due to a lack of initial data the initial conditions are just to set $\mathbf{u}|_{t=0} = 0$ and $\eta|_{t=0} = 0$. The transient part of the solution is sufficiently damped as we start our tidal analysis after 25 cycles of iteration.

2.3. Non-conforming FE models—NCLF, NCAB and NCSI

NCLF, NCAB and NCSI models use a $P_1^{NC} - P_1$ FE discretization (Hanert et al., 2005; Le Roux et al., 2005). The velocity is expanded in non-conforming linear functions associated with edges, and the elevation is expanded in linear functions associated with nodes. The non-conforming element pair $P_1^{NC} - P_1$ is little affected by computational modes (Le Roux et al., 2005, 2007; Le Roux, 2005)