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Beach profile evolution as an inverse problem

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ABSTRACT

Beach evolution models are normally applied in a prognostic fashion, with parameters and boundary conditions estimated from previous experience or other forecasts. Here, we use observations of beach profiles to solve a beach profile evolution equation in an inverse manner to determine model parameters and source function. The data used to demonstrate the method are from Christchurch Bay in Dorset, UK. It was found that there is a significant contribution from diffusive processes to the morphodynamic evolution of the beach profiles and that the development and disappearance of near-shore coastal features such as upper beach berms and inter- and sub-tidal bars are well captured by the source function in the governing equation.

1. Introduction

Predicting morphological changes in coastal systems is a non-trivial task due to the complexity of the underlying physical processes involved and the sensitivity of the system behaviour to natural variability. Interaction between system components and dynamic forces behind its evolution spans a wide range of time scales. The uncertainty of deterministic predictions of dynamic forces beyond certain time scales and ambiguity of non-linear interactions between the system and dynamic forces makes medium to long-term morphodynamic predictions of coastal systems extremely difficult.

Morphodynamic predictions of coastal systems are based on two modelling approaches (De Vriend et al, 1993; De Vriend, 2003). The first approach is the use of process models based on two or three dimensional hydrodynamic models combined with sediment transport and morphodynamic modules (van Rijn et al., 2003; Roelvink et al., 2001). These models are a valuable tool for assessing local, short-term morphodynamic changes in a beach, but have inherent limitations due to the lack of knowledge of sediment transport processes and their linkage to hydrodynamics. Uncertainties in the predictions are amplified by treating sediment with a range of grain sizes. Further, numerical predictions can exhibit great sensitivity to the initial conditions. This is due not just to the accumulation of numerical rounding errors in the computations required to solve the equations but also due to nonlinearity of many coastal systems that may induce chaotic behaviour. The second group of models have been termed ‘behaviour-oriented models’. These models are designed to overcome the difficulties arising out of application of process based modelling (Cowell et al., 1992, 1994; Dean, 1991; Stive and de Vriend, 1995; Reeve and Fleming, 1997). The aim of behaviour-oriented models is to reproduce the qualitative behaviour of beach morphology using a simplified governing equation, parameterising the key processes. The governing equations are rarely derived from first principles; rather, they are defined along the lines of physical arguments. This and the parameterisation of processes are both the strength and potential weakness of such methods.

Diffusion type formulations have been used in the past to model long-term coastal and estuarine morphodynamic behaviour. It is important to note that this type of equations that have been applied to coastal morphology have not derived rigorously from basic process equations but are selected because their solutions qualitatively exhibit the behaviour of the application (Pelnard-Considere, 1956; Reeve and Spivack, 1994; Stive and De Vriend, 1995; Reeve and Spivack, 2000; Hansen et al., 2003; Karunarathna et al., 2008). The success of these models depends on the identification of fundamental parameters as the space and time varying coefficients of a simplified dynamic equation. In the application of a diffusion type model to beach profile change, collective changes to beach profile morphology including development, disappearance and evolution of near-shore morphological features and flattening and steepening of the profile, which are driven by external forces are all included in a source function, which is reproduced based on field evidence.

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In this paper we present a technique for the determination and recovery of the diffusion coefficient and an unknown source function in an advection-diffusion type governing equation for long-term beach profile evolution. The diffusion coefficient is derived as a problem of error minimisation and the source function is recovered as the solution of an inverse problem, using measurements of historic cross-shore beach profiles.

In Section 2 of the paper, the governing equation of the model and the methodology used to derive the diffusion coefficient and source function are presented and explained. The field site and the historic data used to demonstrate the methodology are presented in Section 3. Results are presented and discussed in Section 4, and the paper finishes with conclusions in Section 5.

2. Formulation of the model

This section of the paper describes the simplified beach profile evolution model and the method of recovery of the diffusion coefficient and the source function.

Following the approach suggested by Stive and de Vriend (1995) we take the governing equation for the evolution of beach profiles, relative to a fixed reference level, as a form of advection-diffusion equation:

$$\frac{\partial h(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(K(x)\frac{\partial h(x,t)}{\partial x}\right) + S(x,t) \tag{1}$$

where $h(x,t)$ is the cross-shore beach profile depth measured relative to a fixed reference line, x is the cross-shore position, $K(x,t)$ and $S(x,t)$ are space and time dependent diffusion coefficient and an unknown external source function respectively. Fig. 1 shows the schematics of the model.

There are two unknowns to be resolved in the governing equation, Eq. (1): the diffusion coefficient $K(x,t)$ and the time and space-varying source function $S(x,t)$. Once these unknowns are found (up to a fixed time t_1 say), the governing equation can be used to predict future evolution of beach profiles. Finding suitable values for the diffusion coefficient and the source function is the key element to the success of the model.

The spatial variation of the diffusion coefficient allows us to represent the variation of morphological time scale with cross-shore position. All information about the typical site climate, sediment characteristics and short-term dynamics are assumed to be summarised in $K(x,t)$. All other natural inputs from climate change and human induced inputs are included in the source function $S(x,t)$.

Next we perform a 'Reynold's expansion', writing the profile depth $h(x,t)$, the diffusion coefficient K and the source function S as the sum of their time averaged values and a time varying component as follows:

$$h(x,t) = \overline{h}(x) + h'(x,t) \tag{2}$$

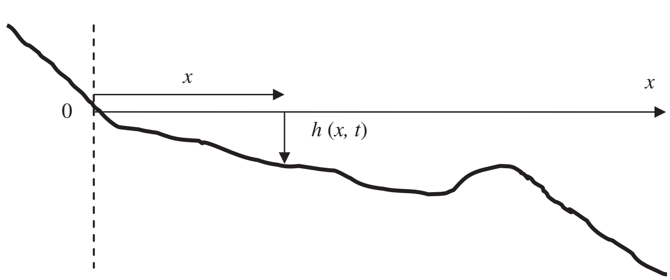

Fig. 1. Schematics of profile evolution model.

$$K(x,t) = \overline{K}(x) + K'(x,t) \tag{3}$$

$$S(x,t) = \overline{S}(x) + S'(x,t) \tag{4}$$

where an over-bar denotes the time averaged components and a prime denotes the time varying residuals.

Then, Eq. (1) can be re-written as

$$\frac{\partial[\overline{h}(x)+h'(x,t)]}{\partial t} = \frac{\partial}{\partial x}\left([\overline{K}(x)+K'(x,t)]\frac{\partial[\overline{h}(x)+h'(x,t)]}{\partial x}\right) + \overline{S}(x) + S'(x,t) \tag{5}$$

or

$$\frac{\partial[\overline{h}(x)+h'(x,t)]}{\partial t} = \frac{\partial}{\partial x}\left(\overline{K}(x)\frac{\partial h(x,t)}{\partial x}\right) + \overline{G}(x) + G'(x,t) \tag{6}$$

where we have written

$$G(x,t) = \frac{\partial}{\partial x}\left(K(x,t)\frac{\partial[h(x,t)]}{\partial x}\right) + S(x,t) \tag{7}$$

We assume that the time average is taken over a sufficiently long period that for any variable x, $\bar{x}'=0$, $\partial\bar{x}/\partial t \approx 0$ and that to a first approximation $\bar{S} \approx 0$.

Then,

$$\overline{G}(x) = \overline{\frac{\partial}{\partial x}\left(K(x,t)\frac{\partial[h(x,t)]}{\partial x}\right)} \tag{8}$$

Taking the time average of Eq. (6) gives:

$$0 = \frac{\partial}{\partial x}\left(\overline{K}(x)\frac{\partial\overline{h}(x,t)}{\partial x}\right) + \overline{G}(x,t) \tag{9}$$

In an analogy to the Reynolds' stresses of turbulent fluid flow, $\bar{G}(x)$ may be considered to be a turbulent morphodynamic stress. As a first order approximation we take these stresses to be zero.

Eq. (9) is then solved for time averaged component of the beach profile,

$$\frac{\partial}{\partial x}\left(\overline{K}(x)\frac{\partial\overline{h}(x)}{\partial x}\right) = 0 \tag{10}$$

The solution of which gives

$$\overline{K}(x) = \frac{\alpha}{(\partial\overline{h}(x)/\partial x)} \tag{11}$$

where α is a constant of integration. $(\partial\bar{h}(x)/\partial x)$ is the gradient of the mean cross-shore beach profile, which may be calculated from the measurements of the beach profiles, and must not be equal to zero anywhere in the range of x considered.

The physical interpretation of Eq. (11) is quite straightforward. To maintain a steep beach the mean diffusion coefficient must be small. Conversely, a large value of the diffusion coefficient corresponds to a gently sloping beach. This accords with the observation that gravel beaches are generally steep (with material that has a relatively slow rate of movement) while fine sand beaches, composed of highly mobile sediment, adopt a gentler incline.

2.1. Determination of time averaged diffusion coefficient

In any application it is understood that a time history of beach profile measurements is available. From these, it will be possible to estimate the mean beach profile and hence it's gradient. This can be used in Eq. (11) as a known quantity. However, there are two unknowns: $\bar{K}(x)$ and α. To solve Eq. (11) for $\bar{K}(x)$ a value for α must be specified. Rather than select a value we adopt a procedure similar to that used by Reeve & Fleming (1997). Let x_i ($i=1, 2, \ldots, N$) denote the x-coordinates for which the average beach profile $\bar{h}(x)$