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# Monthly variation of some parameters about internal solitary waves in the South China sea

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ABSTRACT

In this paper, by non-dimensional analysis, it is found that finite-depth theory is more appropriate to the study of internal solitary waves (ISWs) in the South China Sea (SCS) than shallow-water theory. The 1-degree grid data of monthly mean temperature and salinity data at standard levels in the SCS are used to solve the linearized vertical eigenvalue problem. The nonlinear parameter and the wave phase speed are computed, then the nonlinear phase speed and the characteristic half-width of ISWs are calculated respectively by two different theories to investigate the difference between these two parameters in the SCS. The nonlinearity is the strongest near the continental slope of the SCS or islands where the bottom topography changes sharply, it is stronger in summer than that in winter; it increases (decreases) as pycnocline depth deepens (shallows), stratification strengthens (weakens) and pycnocline thickness thins (thickens). The nonlinear wave phase speed and the characteristic half-width are the largest in deep sea area, they then reduce peripherally in shallower water. The nonlinear wave phase speed in the SCS changes slightly with time, but the characteristic half-width changes somewhat larger with time. In most of the SCS basin, the nonlinear wave phase speed derived from shallow-water theory is very close to that derived from finite-depth theory, but the characteristic half-width derived from shallow-water theory is about 0.2–0.6 times larger than that derived from finite-depth theory. The ISW induced horizontal current velocity derived from shallow-water theory is larger than that derived from finite-depth theory. Some observed and numerical modeled ISW characteristic half-widths are compared with those derived from shallow-water and finite-depth theories, respectively. It is shown that, the characteristic half-widths derived from finite-depth theory agree better with observational and numerical modeled results than those derived from shallow-water theory in most cases, finite-depth theory is more applicable to the estimation of ISW characteristic half-widths in the northern SCS. It is also suggested that, to derive the precise ISW parameters in further study, the physical non-dimensional ratios which are related with ISW characteristic half-width, amplitude, thermocline and water depths should be calculated, so that an appropriate theory can be chosen for estimation.



## 1. Introduction

Internal solitary waves (ISWs) are the nonlinear large amplitude waves existing in the oceanic pycnocline. Over the past several decades, the combination of in situ and remote sensing observations has demonstrated that ISWs are ubiquitous features of regional oceans. As the waves propagate down below the pycnocline, they evolve due to the changing topography and stratification, cause unusually strong underwater currents that may bring about large load on deep-sea drilling rigs or platforms (Cai et al., 2003), and result in significant diapycnal mixing (Xu et al., 2012) that is important for a number of coastal processes and biological production (Holligan et al., 1985). Thus, it is natural to ask, can we give a good estimation of the wave characteristics, such as the wave speed and the characteristic half-width of observed ISWs? What kinds of environmental factors influence these wave characteristics? To well understand the characteristic parameters of ISWs, three regimes of weakly nonlinear internal wave theories, including shallow-water theory (Benjamin, 1966; Benney, 1966), deep-water theory (Benjamin, 1967; Ono, 1975) and finite-depth theory (Joseph, 1977; Kubota et al., 1978), have been developed. The analytical work and underlying scaling assumptions can be categorized as follows (e.g., Apel et al., 1985; Liu et al., 1985),

(I) Shallow-water theory (Benjamin, 1966; Benney, 1966)

$$\frac{L}{H_0} \gg 1,\ \frac{h}{H_0} \sim O(1),\ \frac{\eta_0 L^2}{H_0^3} \sim O(1) \tag{1}$$

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(II) Deep-water theory (Benjamin, 1967; Ono, 1975)

$$\frac{L}{H_0}\to 0,\ \frac{L}{h}\gg 1,\ \frac{\eta_0 L}{h^2}\sim O(1) \tag{2}$$

(III) Finite-depth theory (Joseph, 1977; Kubota et al., 1978)

$$\frac{L}{h}\gg 1,\ \frac{h}{H_0}\ll 1,\ \frac{L}{H_0}\sim O(1),\ \frac{\eta_0 L}{h^2}\sim O(1) \tag{3}$$

where $L$ is the characteristic half-width of ISW, $h$ the length scale associated with thermocline depth, $\eta_0$ the maximum ISW amplitude, $H_0$ the depth of ocean.

E.g., for ISWs in shallow-water, the amplitude $\eta$ satisfies the KdV equation (Apel et al., 1995; Whitham, 1974),

$$\frac{\partial \eta}{\partial t}+c_1\frac{\partial \eta}{\partial x}+\alpha\eta\frac{\partial \eta}{\partial x}+\beta_1\frac{\partial^3 \eta}{\partial x^3}=0 \tag{4}$$

where, $x$ is the horizontal coordinate, and the nonlinear parameter $\alpha$ is,

$$\alpha=\frac{3c_1\int_{-H_0}^{0}(dW_1/dz)^3 dz}{2\int_{-H_0}^{0}(dW_1/dz)^2 dz} \tag{5}$$

while the dispersion parameter $\beta_1$ is,

$$\beta_1=\frac{c_1\int_{-H_0}^{0}{W_1}^2 dz}{2\int_{-H_0}^{0}(dW_1/dz)^2 dz} \tag{6}$$

here, the dimensionless modal function $W(z)$ with its maximum value normalized to unity satisfies the following boundary value problem (Gill, 1982),

$$\frac{d^2W}{dz^2}+\frac{W}{c^2}N^2(z)=0 \tag{7}$$

with $W(0)=W(-H_0)=0$, where $N(z)=(-(g/\rho)(d\rho/dz))^{1/2}$ is the buoyancy frequency, $\rho$ is the water density, $g$ the gravity acceleration, $z$ the vertical coordinate, and $c_1$ and $W_1$ are the linear mode-1 phase speed and eigenfunction, respectively.

The steady solution of the KdV-typed depression ISW (Fig. 1) to Eq. (4) has the form (Apel et al., 1995),

$$\eta=-\eta_0 \mathrm{sech}^2(\varphi_1) \tag{8}$$

here, $\varphi_1=(x-V_1 t)/L_1$, and the nonlinear ISW speed $V_1$ is,

$$V_1=c_1+\alpha\eta_0/3 \tag{9}$$

and the characteristic half-width $L_1$ is,

$$L_1=\sqrt{12\beta_1/\alpha\eta_0} \tag{10}$$

From the linearized kinematic boundary condition $w=(\partial\eta/\partial t)$ and the continuity equation for incompressible flow $\partial u/\partial x=-(\partial w/\partial z)$, the ISW induced vertical and horizontal current velocities $w_1$ and $u_1$

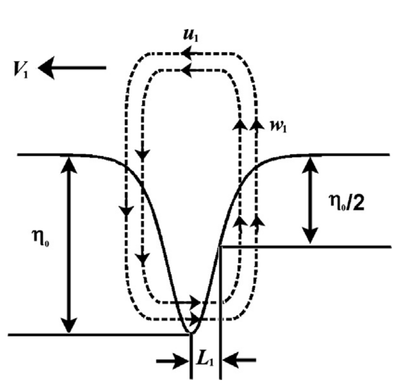


**Fig. 1.** A sketch of the KdV-typed depression ISW (where $L_1$ is the characteristic half-width, $\eta_0$ is the amplitude, $V_1$ is the nonlinear ISW speed, $w_1$ and $u_1$ are the induced vertical and horizontal current velocities).

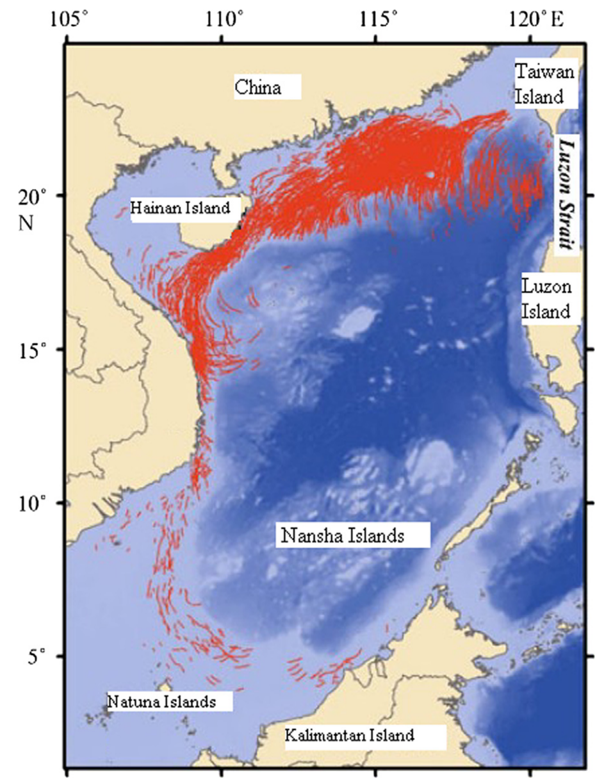


**Fig. 2.** ISW distribution in the SCS from remote sensing satellites (adapted from Wang et al., 2012).

are,

$$w_1=-\frac{2\eta_0 V_1}{L_1}W_1 \mathrm{sech}^2(\varphi_1)\tanh(\varphi_1) \tag{11}$$

$$u_1=-\eta_0 V_1\frac{dW_1}{dz}\mathrm{sech}^2(\varphi_1) \tag{12}$$

According to Eqs. (4)–(12), KdV-typed ISW retains its shape and speed during its propagation due to a balance between nonlinear and dispersive effects. The nonlinear parameter $\alpha$ and the dispersion parameter $\beta_1$ are known as environmental parameters and incorporate the effects of buoyancy, current, current shear, and water depth via their effects on the eigenfunction profiles, $W(z)$. The characteristic half-width $L_1$ is determined by $\eta_0$, $\alpha$ and $\beta_1$, and it is also related with the ISW induced vertical current velocity $w_1$. The nonlinear ISW speed $V_1$, the ISW induced vertical and horizontal current velocities $w_1$ and $u_1$ are all related with $\alpha$. Note that $\beta_1$ is always positive from Eq. (6), whilst $\alpha$ may change its sign as the ISW passes through a location of critical depth which is corresponding to the case of equal layer depths for two-layer system (e.g., Liu, 1988; Apel et al., 1995). From Eq. (10), $\alpha\eta_0\geq 0$, thus if $\alpha$ is negative (positive), so will be $\eta_0$, i.e., the ISW will be a depression (elevation) one. So, the nonlinear ISW speed $V_1$ increases with $\alpha$. If $\beta_1$ increases (decreases), it indicates the decompression (compression) of ISWs. $\alpha$ is very sensitive to stratification, whereas $\beta_1$ is weakly dependent on the detail of stratification, and the ratio of $\alpha$ to $\beta_1$, the Ursell number, affects the nature of waveform, e.g., a solitary waveform occurs when the Ursell number is large, and a linear sinusoidal waveform occurs when the Ursell number is small (Lee and Beardsley, 1974; Holloway and Pelinovsky, 2002; Yang et al., 2009). Therefore, the seasonal change in stratification will affect the nonlinear and dispersion parameters, the characteristics of ISWs and the associated circulation largely.