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An approximate solution of wave-modified Ekman current for gradually varying eddy viscosity☆

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ABSTRACT

An approximate steady solution of the wave-modified Ekman current is presented for gradually varying eddy viscosity by using the WKB method with the variation of parameters technique. The parameters involved in the solution can be determined by the two-dimensional wavenumber spectrum of ocean waves, wind speed, the Coriolis parameter and the densities of air and water. The solution reduces to the exact solution when the eddy viscosity is taken as a constant. As illustrative examples, for a fully developed wind-generated sea with different wind speeds and a few proposed gradually varying eddy viscosities, the current profiles calculated from the approximate solutions are compared with those of the exact solutions or numerical ones by using the Donelan and Pierson wavenumber spectrum, the WAM wave model formulation for wind input energy to waves, and wave energy dissipation converted to currents. It is shown that the approximate solution presented has an elegant form and yet would be valid for any given gradually varying eddy viscosity. The applicability of the solution method to the real ocean is discussed following the comparisons with published observational data and with the results from a large eddy simulation of the Ekman layer.

1. Introduction

The effects of wind-driven surface gravity waves on ocean surface currents have been recognized to play a crucial role for scientific and engineering applications. For example, they are important in the interpretation of satellite images and the impact of surface currents on satellite-derived wind estimates (Quilfen et al., 2001; Kelly et al., 2001), the correction of biases in radar-derived surface currents (Chapron et al., 2005), sea ice drift (Tang and Gui, 1996), various biological processes such as the drift of fish eggs and larvae (Brickman and Frank, 2000; Reiss et al., 2000), environmental loading on offshore structures (Farmer et al., 1995), the dispersion and drift of oil and other pollutants (Leibovich, 1997a, 1997b; Morinta et al., 1997), hurricane intensities (Emanuel, 1999; Andreas and Emanuel, 2001) and climate (Tang et al., 2002).

Waves grow and evolve in space and time, interacting with ocean currents and reflecting the structure and development of the wind stress fields that generate them. As they experience wave breaking and dissipation, momentum passes from waves into ocean currents. Using irrotational theory for wave growth and wave breaking, Weber (1983), Weber and Melsom (1993) and Melsom (1996) investigated the conversion of the wave pseudo-momentum to momentum of the mean Eulerian current from wave dissipation caused by the eddy viscosity. Jenkins (1986, 1987a, 1987b, 1989) developed the corresponding formulation based on an ocean spectral wave model. Perrie et al. (2003) and Tang et al. (2007) coupled the formulation of Jenkins (1987a, 1987b, 1989) to an ocean model to investigate the impact of waves on surface currents. They showed that the wave effect could increase the surface current by as much as 40%.

Recent studies show that the influence of the surface wave motion via the Stokes drift and mixing is fundamental to understanding the observed Ekman current profiles (Lewis and Belcher, 2004; Polton et al., 2005; Rascle et al., 2006), although real Ekman currents are the products of a host of interrelated factors, including wind stress, surface wave motion, surface heating and so on. Following the approach of Jenkins (1987a, 1987b, 1989) and Perrie et al. (2003), Song (2009) presented the steady analytic solutions for modified Ekman equations including random surface wave effects when the eddy viscosity coefficient is, respectively,

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taken as depth-independent and proportional to depth. The effects of random waves on the classical Ekman current are then studied by comparing the solutions including waves to those with no waves. In this paper, an approximate solution of the model used by Song (2009) is presented for gradually varying eddy viscosities using the WKB method, and this WKB solution is studied and compared with the exact and numerical solutions of the model for a few proposed eddy viscosities. Possible applications of the solution method to the real ocean are discussed.

2. Basic equations and boundary conditions

When the effects of random surface waves are considered, the steady wind-driven Ekman horizontal current satisfy the following modified Ekman equation (Jenkins, 1989; Rascle et al., 2006; Tang et al., 2007; Song, 2009)

$$A_v(z)\frac{\partial^2 U_{WE}(z)}{\partial^2 z}+\frac{\partial A_v(z)}{\partial z}\left[\frac{\partial U_{WE}(z)}{\partial z}\right]-ifU_{WE}(z)=ifU_s(z)+T_{wds}(z), \tag{1}$$

where $U_{WE}=u_{WE}+iv_{WE}$ is the complex horizontal velocity in the x–y plane, $i=\sqrt{-1}$, f is the Coriolis parameter. The horizontal coordinate axes are fixed on the still water level with $z=0$, the z-axis is along the vertical direction with positive direction upwards, $A_v(z)$ is the vertical eddy viscosity coefficient, $U_s=u_s+iv_s$ is the complex Stokes drift, and T_{wds} is the wave-induced momentum transfer from waves to the mean flow due to dissipation of wave energy. The velocity $\mathbf{u}_{WE}=(u_{WE},v_{WE})$ discussed here is the quasi-Eulerian current, which is equal to the Lagrangian mean current minus the Stokes drift and can be understood as the Eulerian-mean current as stated by Jenkins (1987a; 1989) and Perrie et al. (2003).

The surface boundary condition for the modified Ekman current is

$$A_v\frac{\partial U_{WE}}{\partial z}=\frac{\tau_a}{\rho_w}-\frac{\tau_{in}}{\rho_w},\quad z=0, \tag{2}$$

where ρ_w is the water density, τ_{in} is the reduction of wind stress due to wave generations, $\tau_a=\tau_{ax}+i\tau_{ay}$ is the complex wind stress, computed from surface wind field $\mathbf{U}_{10}$ at 10 m height,

$$\boldsymbol{\tau}_a=(\tau_x,\tau_y)=\rho_a C_d|U_{10}|\mathbf{U}_{10}, \tag{3}$$

ρ_a is the air density, and C_d is the air-sea drag coefficient, which is related to U_{10} by the following relation (Wu, 1982):

$$C_d=(0.8+0.065U_{10})\times 10^{-3}. \tag{4}$$

The lower boundary condition is taken as

$$U_{WE}\to 0,\quad (z\to-\infty). \tag{5}$$

τ_{in} and T_{wds} can be estimated by the source terms from a directional spectral wave prediction model that act to transfer momentum from the wave field to the current as follows (Jenkins, 1989, also see Tang et al., 2007):

$$\tau_{in}=\tau_{inx}+i\tau_{iny}=\rho_w\int\left(\frac{\omega}{k}\right)KS_{in}(k,\theta)dk\,d\theta, \tag{6}$$

$$T_{wds}=T_{wdsx}+iT_{wdsy}=2\int\omega Ke^{2kz}S_{ds}(k,\theta)dk\,d\theta, \tag{7}$$

where ω is the angular frequency in rad/s, k is the moduli of the horizontal wavenumber vector $\mathbf{k}=(k_x,k_y)=(k\cos\theta,k\sin\theta)$, their relationship is given by the dispersion relation $\omega^2=gk$, θ is the angle between the wave vector and the x-axis, $K=k_x+ik_y$, $S_{in}(k,\theta)$ is the wind input energy to waves, $S_{ds}(k,\theta)$ is the wave energy lost by wave dissipation mechanisms as represented in third-generation WAM-type models (Hasselmann et al., 1988; Komen et al., 1994).

Stokes drift $\mathbf{u}_s$ may be expressed as (Kenyon, 1969; Huang, 1971)

$$\mathbf{u}_s=2\int\omega\mathbf{k}e^{2kz}E(k,\theta)dk\,d\theta, \tag{8}$$

where $E(k,\theta)$ is the directional wavenumber spectrum of surface waves.

3. Approximate solutions

Eq. (1) is a linear inhomogeneous ordinary differential equation with variable coefficients if the eddy viscosity $A_v(z)$ is specified. Thus, it can then be solved by ordinary differential equation methods. Following Berger and Grisogono (1998), studying the Ekman atmospheric boundary layer, an approximate solution to the inhomogeneous problem (1) can be found with the variation of parameters technique, provided that an approximate solution to the homogeneous problem of (1) exists. If two independent approximate solutions to homogeneous problem of Eq. (1) are given by $\Phi_1(z)$ and $\Phi_2(z)$, the general solution of Eq. (1) is given by

$$U_{WE}(z)=[\hat{c}_1\Phi_1(z)+\hat{c}_2\Phi_2(z)]+[c_1(z)\Phi_1(z)+c_2(z)\Phi_2(z)], \tag{9}$$

where $\hat{c}_1$ and $\hat{c}_2$ are constants, $\hat{c}_1\Phi_1(z)+\hat{c}_2\Phi_2(z)$ represents the complementary solution, and $c_1(z)\Phi_1(z)+c_2(z)\Phi_2(z)$ is the particular solution with

$$c_1(z)=-\int_0^z\frac{\Phi_2(z')\eta(z')}{F(z')}dz', \tag{10}$$

$$c_2(z)=\int_0^z\frac{\Phi_1(z')\eta(z')}{F(z')}dz', \tag{11}$$

where

$$F(z')=A_v(z')\left[\Phi_1(z')\Phi_2'(z')-\Phi_1'(z')\Phi_2(z')\right], \tag{12}$$

and $\eta(z')=ifU_s(z')+T_{wds}(z')$ is the inhomogeneous part of Eq. (1).

To find the homogeneous solutions $\Phi_1(z)$ and $\Phi_2(z)$, following Grisogono (1995) in a study of the atmospheric Ekman layer, the WKB method is applied. The first and third terms in the left of Eq. (1) represent the control behavior, which is the basis of the WKB analysis, while the second term is identified with the modifications due to departures of A_v from constant. The WKB expansion for U_{WE} can be expressed in the form

$$U_{WE}\propto\exp\{(S_0+S_1\varepsilon+S_2\varepsilon^2+\cdots)/\varepsilon\}. \tag{13}$$

The WKB approach can provide a good approximate solution, so long as the properties of the medium vary at least slightly slower than the calculated quantities.

Substituting Eq. (13) into Eq. (l), we obtain a set of equations in terms of powers of a presumably small parameter ε (ε has been introduced on account of the above-mentioned balance between the terms and at a later stage it will be equated to unity).

As analyzed by Grisogono (1995), the validity of the WKB method requires that the variable eddy viscosity does not vary too quickly with depth, A_v and f do not change their signs and that:

$$\frac{|S_{n+1}(z)|}{|S_n(z)|}<<1,\quad n=0,\ 1,\ 2,\ldots, \tag{14}$$

over the intervals. Also, if S_N is the last term used in the series,

$$|S_{N+1}(z)|<<1. \tag{15}$$

The first two terms of the expansion are sufficient to give a meaningful, yet simple solution. Solving for S_0 and S_1 yields

$$S_0=\pm(1+i)\sqrt{\frac{f}{2}}\int_0^z\frac{1}{\sqrt{A_v(z')}}dz', \tag{16}$$