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Routing strategies for underwater gliders

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ABSTRACT

Gliders are autonomous underwater vehicles that achieve long operating range by moving at speeds comparable to those of, or slower than, typical ocean currents. This paper addresses routing gliders to rapidly reach a specified waypoint or to maximize the ability to map a measured field, both in the presence of significant currents. For rapid transit in a frozen velocity field, direct minimization of travel time provides a trajectory "ray" equation. A simpler routing algorithm that requires less information is also discussed. Two approaches are developed to maximize the mapping ability, as measured by objective mapping error, of arrays of vehicles. In order to produce data sets that are readily interpretable, both approaches focus sampling near predetermined "ideal tracks" by measuring mapping skill only on those tracks, which are laid out with overall mapping skill in mind. One approach directly selects each vehicle's headings to maximize instantaneous mapping skill integrated over the entire array. Because mapping skill decreases when measurements are clustered, this method automatically coordinates glider arrays to maintain spacing. A simpler method that relies on manual control for array coordination employs a first-order control loop to balance staying close to the ideal track and maintaining vehicle speed to maximize mapping skill. While the various techniques discussed help in dealing with the slow speed of gliders, nothing can keep performance from being degraded when current speeds are comparable to vehicle speed. This suggests that glider utility could be greatly enhanced by the ability to operate high speeds for short periods when currents are strong.

1. Introduction

Underwater gliders are designed to have long endurance (months) and to navigate autonomously by periodically surfacing for GPS fixes and data transmission. Stommel (1989) advanced the concept and today there are at least three well-tested models (Davis et al., 2002). The operational consequence of designing for endurance is low speed (0.2–0.4 m s^{-1}), comparable to that of ocean currents and much lower than that of strong boundary currents. This severely limits the ability of glider operators to place observations where they want them and raises the question about how to route gliders through velocity fields. That is the topic of this investigation.

The terminology is made clear by considering the simplest case of a vehicle moving at speed q and heading θ (reckoned as in the complex plane, not a compass) through water that has uniform velocity $\mathbf{u}$ with magnitude u and direction ω. The vehicle's velocity over the ground is $\mathbf{U}$ with speed U and direction ϕ, here called the course over the ground (COG). These velocities are illustrated in Fig. 1. Let $u_\phi = u\cos(\omega-\phi)$ be the current component assisting motion along the track and $u_\mathrm{N} = u\sin(\omega-\phi)$ be the current component 90° to the left of the track. Staying on the desired path requires the cross-track component of vehicle velocity $q\sin(\theta-\phi)$ to cancel the cross-track current u_N. The heading θ and speed U made good along the desired path are then

$$\theta = -\arcsin(u_\mathrm{N}/q) + \phi, \quad U = u_\phi + q\sqrt{1-(u_\mathrm{N}/q)^2}. \tag{1}$$

So long as $|u_\mathrm{N}| < q$ the vehicle can stay on the desired track, but the velocity made good decreases as $|u_\mathrm{N}| \to q$. A central question addressed here is about how, in more complex velocity fields, to route a glider to reach a destination as quickly as possible. For steady currents the fastest route is given by an equation with strong parallels to the ray equations for nondispersive wave propagation.

A second question addressed is about how to route gliders, operating singly or in groups in a field of significant currents, to maximize what is learned about a measured field. The reduction in error variance of objective maps is used to measure the "skill" of a particular sampling strategy. If mapping skill were to be the sole criterion, the resulting glider trajectories would be

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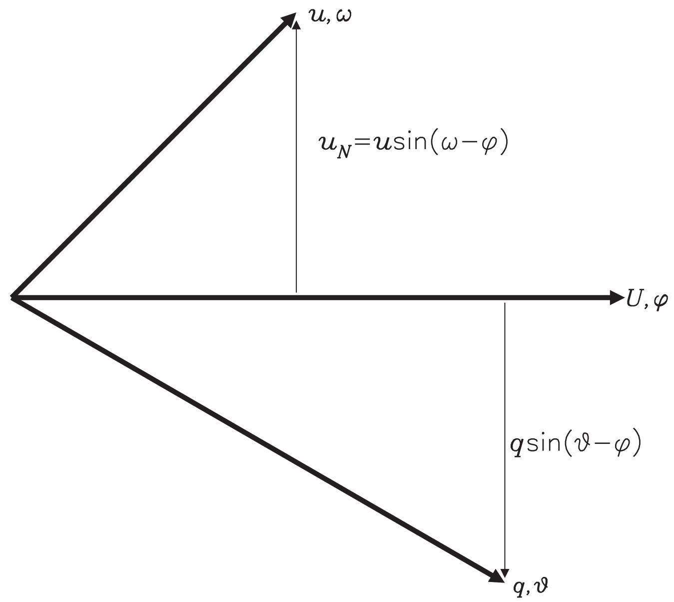

Fig. 1. Illustration of the velocities in glider steering. The vehicle's through-the-water speed and heading are q and θ, the ocean current $\mathbf{u}$ has speed and direction u and $\boldsymbol{\omega}$, and the resultant vehicle velocity over the ground is $\mathbf{U}$ with speed and course U and $\boldsymbol{\phi}$, respectively. Also shown are the cross-track components of through-water vehicle velocity $q\sin(\boldsymbol{\theta}-\boldsymbol{\phi})$ and current velocity $u_N = u\sin(\boldsymbol{\omega}-\boldsymbol{\phi})$ that cancel to give (1). The current component parallel to the track over the ground is $u_{\phi} = u\cos(\boldsymbol{\omega}-\boldsymbol{\phi})$.

so irregular that it would be impossible to interpret them without insertion into a mapping or data assimilation procedure. Optimizing mapping skill on a network of ideal paths pre-selected to yield good observational coverage provides an automated steering procedure for optimizing both interpretability and mapping skill, including coordinating an array of vehicles.

Vehicle speed is central to both transit between points and mapping. Movement is important in mapping because mapping skill, as measured by objective analysis, increases with the number of uncorrelated observations made within a correlation time. The faster a vehicle moves, the more uncorrelated measurements it gathers. Because mapping also depends on making measurements in the right places, there is a tension between maintaining vehicle velocity to keep samples well separated and moving along a useful track in the face of currents.

2. Strategies for fast routing

Because gliders are so slow, currents have a first-order impact on vehicle speed. Here we address strategies to minimize the time required to navigate between two points through a current field. The formalism is derived in the same way Fermat's principle is used to develop ray-tracing equations for sound or light propagation (Pierce, 1989). Let the steady water velocity be $\mathbf{u}(\mathbf{x})$ while q and θ are the glider's speed through the water and its heading, respectively. The coordinates of a glider trajectory are $x(\lambda)$, $y(\lambda)$ where λ is an as yet undefined label of position along the path between endpoints $\mathbf{x}_A$ and $\mathbf{x}_B$. Finally, let $1/s$ be the speed of the vehicle along the path, equal to U in (1) above. The fastest route from $\mathbf{x}_A$ to $\mathbf{x}_B$ is the path that minimizes the travel time

$$T_{\mathrm{T}} = \int_A^B \mathrm{d}\lambda \frac{\mathrm{d}t}{\mathrm{d}\lambda} = \int_A^B \mathrm{d}\lambda\, s\eta, \tag{2}$$

where $\eta = (\mathrm{d}\ell/\mathrm{d}\lambda) = \sqrt{(\mathrm{d}x/\mathrm{d}\lambda)^2 + (\mathrm{d}y/\mathrm{d}\lambda)^2}$ and ℓ is the arc length.

2.1. Rays for nondispersive wave propagation

In sound or light propagation, slowness s is a function of position $\mathbf{x}$ and the variation of travel time δT_{T} resulting from variations $\delta x(\lambda)$, $\delta y(\lambda)$ of the path is

$$\delta T_{\mathrm{T}} = \int_A^B \mathrm{d}\lambda \left\{ \eta \left[\frac{\partial s}{\partial x}\delta x + \frac{\partial s}{\partial y}\delta y \right] + \frac{s}{\eta}\left[\frac{\mathrm{d}x}{\mathrm{d}\lambda}\frac{\mathrm{d}\,\delta x}{\mathrm{d}\lambda} + \frac{\mathrm{d}y}{\mathrm{d}\lambda}\frac{\mathrm{d}\,\delta y}{\mathrm{d}\lambda} \right] \right\}. \tag{3}$$

With the conditions $\delta\mathbf{x} = 0$ at $\mathbf{x}_A$ and $\mathbf{x}_B$, (3) is easily integrated by parts to

$$\delta T_{\mathrm{T}} = \int_A^B \mathrm{d}\lambda \left\{ \left[\eta\frac{\partial s}{\partial x} - \frac{\partial}{\partial\lambda}\left(\frac{s}{\eta}\frac{\mathrm{d}x}{\mathrm{d}\lambda}\right) \right]\delta x + \left[\eta\frac{\partial s}{\partial y} - \frac{\partial}{\partial\lambda}\left(\frac{s}{\eta}\frac{\mathrm{d}y}{\mathrm{d}\lambda}\right) \right]\delta y \right\}. \tag{4}$$

If λ is taken as the arc length itself, then $\eta = 1$ and paths that extremize T_{T} obey

$$\frac{\partial}{\partial\lambda}\left(s\frac{\mathrm{d}x}{\mathrm{d}\lambda}\right) = \frac{\partial s}{\partial x}, \qquad \frac{\partial}{\partial\lambda}\left(s\frac{\mathrm{d}y}{\mathrm{d}\lambda}\right) = \frac{\partial s}{\partial y}. \tag{5}$$

When λ is the arc length, $\mathrm{d}x/\mathrm{d}\lambda = \cos\phi$ and $\mathrm{d}y/\mathrm{d}\lambda = \sin\phi$, the degenerate equation (5) becomes

$$s\frac{\mathrm{d}\phi}{\mathrm{d}\lambda} = \hat{\boldsymbol{\lambda}} \times \nabla s, \tag{6}$$

where $\hat{\boldsymbol{\lambda}}$ is a unit vector parallel to the path.

2.2. Ray equations for fast routes

For a vehicle moving through a current $\mathbf{u}(\mathbf{x})$, (1) shows the vehicle speed to be

$$1/s(\mathbf{x},\phi) = \mathbf{u}(\mathbf{x})\cdot\hat{\boldsymbol{\lambda}} + \sqrt{q^2 - |\mathbf{u}(\mathbf{x})\times\hat{\boldsymbol{\lambda}}|^2}. \tag{7}$$

Here s depends on position, as it does in (3), and also on the direction of the path:

$$\phi = \arctan\left(\frac{\mathrm{d}y/\mathrm{d}\lambda}{\mathrm{d}x/\mathrm{d}\lambda}\right), \tag{8a}$$

which is the COG as in (1). The travel time is still given by (2) but its variation now includes, within the integral, the term $(\partial s/\partial\phi)\delta\phi$ not found in (3). This reflects the effect of COG on vehicle speed. The variation of ϕ from (8a) is

$$\delta\phi = \frac{(\mathrm{d}x/\mathrm{d}\lambda)^2}{(\mathrm{d}x/\mathrm{d}\lambda)^2 + (\mathrm{d}y/\mathrm{d}\lambda)^2}\left[\frac{\delta(\mathrm{d}y/\mathrm{d}\lambda)}{\mathrm{d}x/\mathrm{d}\lambda} - \frac{\mathrm{d}y/\mathrm{d}\lambda}{(\mathrm{d}x/\mathrm{d}\lambda)^2}\delta(\mathrm{d}x/\mathrm{d}\lambda) \right] \tag{8b}$$

and the variation of T_{T} is

$$\begin{aligned}\delta T_{\mathrm{T}} = \int_A^B \mathrm{d}\lambda \Bigg\{ &\eta\,\nabla s\cdot\delta\mathbf{x} + \frac{s}{\eta}\left[\frac{\mathrm{d}x}{\mathrm{d}\lambda}\frac{\mathrm{d}\,\delta x}{\mathrm{d}\lambda} + \frac{\mathrm{d}y}{\mathrm{d}\lambda}\frac{\mathrm{d}\,\delta y}{\mathrm{d}\lambda} \right] \\ &+ \frac{1}{\eta}\frac{\partial s}{\partial\phi}\left[\frac{\mathrm{d}x}{\mathrm{d}\lambda}\frac{\mathrm{d}\,\delta y}{\mathrm{d}\lambda} - \frac{\mathrm{d}y}{\mathrm{d}\lambda}\frac{\mathrm{d}\,\delta x}{\mathrm{d}\lambda} \right] \Bigg\},\end{aligned} \tag{9a}$$

$$\frac{\partial s}{\partial\phi} = s^2\hat{\mathbf{z}}\cdot(\mathbf{u}\times\hat{\lambda})\left[1 + \frac{\mathbf{u}\cdot\hat{\boldsymbol{\lambda}}}{\sqrt{q^2 - |\mathbf{u}\times\hat{\boldsymbol{\lambda}}|^2}} \right]. \tag{9b}$$

If, as in the wave propagation case, (9a) is integrated by parts with $\delta\mathbf{x} = 0$ at $\mathbf{x}_A$ and $\mathbf{x}_B$ to convert $\mathrm{d}\,\delta x/\mathrm{d}\lambda$ to δx, the result is

$$\begin{aligned}\delta T_{\mathrm{T}} = \int_A^B \mathrm{d}\lambda \Bigg\{ &\delta x\left[\eta\frac{\partial s}{\partial x} - \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{s}{\eta}\frac{\mathrm{d}x}{\mathrm{d}\lambda} - \frac{1}{\eta}\frac{\partial s}{\partial\phi}\frac{\mathrm{d}y}{\partial\lambda} \right) \right] \\ &+ \delta y\left[\eta\frac{\partial s}{\partial y} - \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{s}{\eta}\frac{\mathrm{d}y}{\mathrm{d}\lambda} + \frac{1}{\eta}\frac{\partial s}{\partial\phi}\frac{\mathrm{d}x}{\mathrm{d}\lambda} \right) \right] \Bigg\}.\end{aligned} \tag{9c}$$