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A new 3D chaotic system with golden proportion equilibria: Analysis and electronic circuit realization ☆

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ABSTRACT

This article introduces a new three-dimensional quadratic continuous autonomous chaotic system with golden proportion equilibria, which can generate single folded attractor. Some basic dynamical behaviors and the dynamical structure of the new chaotic system are investigated either analytically or numerically. Finally, the chaos generator of the new chaotic system was designed, simulated and experimentally confirmed via a novel electronic circuit design. A good qualitative agreement is illustrated between the simulations and the experimental results. It is convenient to use the new system to purposefully generate chaos in chaos applications. The new system has eight terms, two quadratic nonlinearities and two parameters (a and b). The new chaotic attractor equations have three equilibrium points and more interestingly the equilibrium points have golden proportion values.

1. Introduction

It arose amazement among to most scientists when Lorenz discovered chaos in a simple system of three-dimensional quadratic autonomous ordinary differential equations in 1963 [1]. For nearly 40 years, many simple chaotic flows have been found and further studied within the framework of three-dimensional quadratic autonomous systems. Chen constructed another chaotic system [2], which nevertheless is not topologically equivalent to the Lorenz's [2,3]. This system is the dual to the Lorenz system and similarly has a simple structure [3]. Lü and Chen found the critical new chaotic system [4], which represents the transition between the Lorenz and Chen attractors. For the investigation on generic 3D smooth quadratic autonomous systems, Sprott [5–7] found by exhaustive computer searching about 19 simple chaotic systems with no more than three equilibria. It is very important to note that some 3D autonomous chaotic systems have three particular fixed points: one saddle and two unstable saddle-foci (for example, Lorenz system [1], Chen system [2], Lu system [4]). The other 3D chaotic systems, such as the original Rossler system [8], Diffusionless Lorenz System (DLS) [9] and Burke-Show system [10], have two unstable saddle-foci. Yang and Chen found another 3D chaotic system with three fixed points: one saddle and two stable fixed points [11]. Recently, Yang et al. [12] and Pehlivan and Uyaroglu [13] introduced and analyzed the new 3D chaotic systems with six terms including only two quadratic terms in a form very similar to the Lorenz, Chen, Lu and Yang–Chen systems, but they have two very different fixed points: two stable node-foci. Therefore, they are very interesting to further find out the new dynamics of the system.

There has been increasing interest in exploiting chaotic dynamics in engineering applications, where some attention has been focused on effectively creating chaos via simple physical systems, such as electronic circuits [14–18]. Lately, the pursuit of designing circuits to produce chaotic attractors has become a focal point for electronics engineers, not only because of

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their theoretical interest, but also due to their potential real-world applications [19] in various chaos-based technologies and information systems [19–29].

Motivated by such previous works, this article introduces another novel chaotic system with golden ratio equilibria. In Section 2, the new chaotic system is introduced and dynamical behaviors of the new attractor are investigated. In Section 3, an electronic circuit realization of the proposed chaotic system is presented and examined. Finally, conclusions and discussions are given.

2. A new 3D chaotic system with golden proportion equilibria and its analyses

Following nonlinear autonomous ordinary differential equations are related to the new chaotic system.

$$\begin{aligned} \dot{x} &= y - x - a \cdot z \\ \dot{y} &= x \cdot z - x \\ \dot{z} &= -xy - y + b. \end{aligned} \tag{1.1}$$

The new system has eight terms, two quadratic nonlinearities and two parameters (a and b). Typical parameters are $a = 2$, $b = 1$ or $a = 0.5$, $b = 1$. Let us consider a volume in a certain domain of the state space. For the system (1.1), one has

$$\Delta V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -1 = r,$$

with $r = -1$, where r is a negative value. Dynamical system (1.1) is one dissipative system, and an exponential contraction rate of the system (1.1) is

$$\frac{dV}{dt} = e^r = e^{-1}.$$

In the dynamical system (1.1), a volume element V_0 is apparently contracted by the flow into a volume element $V_0e^{rt} = V_0e^{-t}$ in time t. It means that each volume containing the trajectory of this dynamical system shrinks to zero as $t \to \infty$ at an exponential rate r. Therefore, all this dynamical system orbits are eventually confined to a specific subset that have zero volume, the asymptotic motion settles onto an attractor of the system (1.1).

The new system equations have three equilibrium points as

$$E_1\left(\frac{-a-1+\sqrt{a^2-2a+1+4b}}{2}, \frac{a-1+\sqrt{a^2-2a+1+4b}}{2}, 1\right),$$

$$E_2\left(\frac{-a-1-\sqrt{a^2-2a+1+4b}}{2}, \frac{a-1-\sqrt{a^2-2a+1+4b}}{2}, 1\right),$$

$$E_3\left(0, b, \frac{b}{a}\right).$$

As the variables x, y, z $\in \Re$, this implies that fixed point to exist, $a \neq 0$ and $a^2 - 2a + 1 + 4b > 0$. So, $(a-1)^2 + 4b > 0$, and $a \in \Re$. When $a = 0$, the system has unbounded solutions.

The equilibrium points of the new system are

$$E_1\left(\frac{-3+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}, 1\right), \quad E_2\left(\frac{-3-\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}, 1\right), E_3\left(0, 1, \frac{1}{2}\right) \quad \text{for}\, a = 2,\ \text{for}\, b = 1 \text{values}.$$

More interestingly the equilibrium points have golden proportion values as

$$E_1(-\tau^{-2}, \tau, \tau^0), E_2(-\tau^2, -\tau^{-1}, \tau^0), E_3\left(0, \tau^0, \frac{\tau^0}{2}\right).$$

The famous golden proportion $\tau = \frac{1+\sqrt{5}}{2}$, found often in nature. In the last years, the golden proportion has played an increasing role in modern physical research [30–35].

For the $E_1\left(\frac{-3+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}, 1\right)$ equilibrium point, the Jacobian matrix of system (1.1) is given as follows:

$$J(E_1) = \begin{pmatrix} -1 & 1 & -a \\ z_1 - 1 & 0 & x_1 \\ -y_1 & -x_1 - 1 & 0 \end{pmatrix}. \tag{1.2}$$

Obviously, the characteristic equation about the equilibria E_1 is:

$$\det(\lambda I - J(E_1)) = \lambda^3 + \lambda^2 - 1.618034 \cdot \lambda - 1.618034 \cdot \lambda \cdot a - 5.854102 = 0,$$