Fisheries Research 176 (2016) 86–93

Contents lists available at ScienceDirect

Fisheries Research

journal homepage: www.elsevier.com/locate/fishres

Correct in theory but wrong in practice: Bias caused by using a lognormal distribution to penalize annual recruitments in fish stock assessment models

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ARTICLE INFO

Article history:
Received 2 October 2015
Received in revised form
24 November 2015
Accepted 3 December 2015
Available online 7 January 2016

Keywords:
Likelihood
Recruitment
Assessment
Simulation
Fish

ABSTRACT

Penalties are widely used for a range of parameters while fitting fish stock assessment models. Penalizing annual recruitments for deviating from an underlying mean recruitment is probably the most common. Assuming that recruits are log-normally distributed for the purposes of this penalty is theoretically justifiable. In practice, however, bias may be induced because this distributional assumption includes a term equal to the summation of the log observed data, which in the case of recruitment equals the summation of the log recruitment parameters that are not data. Using simulation, the potential for bias caused by assuming that recruits were log-normally distributed was explored, and results were contrasted with the assumption that log-recruitment was normally distributed, an alternative that avoids the potentially troublesome summation term. Spawning stock biomass (*SSB*) and recruitment were negatively biased, while fishing mortality (*F*) was positively biased under the assumption of log-normally distributed recruitments, and the bias worsened closer to the terminal year. The bias also worsened when the true underlying *F* was low relative to natural mortality, and with domed fishery selectivity. Bias in *SSB*, recruitment, and *F* was nonexistent or relatively small under the assumption that log-recruitment was normally distributed. Distributional assumptions for penalties used in assessment models should be reviewed to reduce the potential for biased estimation. These results also provide further support for simulation testing to evaluate statistical behavior of assessment models.

Published by Elsevier B.V.

1. Introduction

Penalizing the objective function is common practice for estimating parameters in fish stock assessment models (e.g., Butterworth et al., 2003; Punt et al., 2011). Penalties have been applied to dampen the degree of interannual variation of time varying parameters (e.g., selectivity, catchability, growth), prevent unrealistically large changes in selectivity among ages, and control the degree to which annual recruitments deviate from an underlying stock-recruitment curve (Ianelli, 2002; Parma 2002; Maunder and Deriso, 2003; Thorson et al., 2015). At least some applications of penalized likelihood, however, can induce biased parameter estimates (Maunder and Deriso, 2003). Despite the potential for bias, little research has evaluated the performance of stock assessments under various assumptions for the penalty terms.

Perhaps the most ubiquitous application of penalized likelihood in stock assessment is for recruitment parameters (Legault and Restrepo, 1999; Brodziak, 2005; Ebener et al., 2005; Butterworth and Rademeyer, 2008; Methot and Wetzel, 2013). Fish stock assessment models commonly estimate recruitment parameters with an assumption for annual recruitment deviations based on some variation of a normal distribution:

$$\hat{R}_y = \overline{R}_y e^{\epsilon_y}; \epsilon_y \sim \mathrm{N}\left(0, \sigma^2\right)$$

where $\hat{R}_y$ is recruitment in year y, $\overline{R}_y$ is mean recruitment that may be a function of spawning stock (e.g., Beverton–Holt, Ricker), ϵ_y is the annual deviation from the log-scale mean, and σ^2 is the variance of the deviations. $\overline{R}_y$ is also sometimes multiplied by $e^{-\frac{\sigma^2}{2}}$ as a bias correction so that the mean of the log-normally distributed $\hat{R}_y$ equals $\overline{R}_y$, and this bias correction may also vary annually (Methot and Taylor, 2011).

A variety of definitions have been used for the contribution of a recruitment penalty to the overall objective function in statistical catch-at-age models (Table 1), but one important distinction is

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http://dx.doi.org/10.1016/j.fishres.2015.12.002
0165-7836/Published by Elsevier B.V.

Table 1
Statistical distributions and associated negative log likelihoods that have been used as a penalty for the contribution of recruitment to the objective function in fish stock assessments, where n_{rec} is the number of recruitment deviations and other symbols are defined as in the main text.

Assumed distribution	Negative log-likelihood
$\hat{R}_y \sim LN\left(\ln\left(\bar{R}_y\right), \sigma^2\right)$	$n_{rec}\frac{\ln(2\pi)}{2} + \sum_{y=1}^{n_{rec}} \ln\left(\hat{R}_y\right) + n_{rec}\ln(\sigma) + \frac{1}{2}\sum_{y=1}^{n_{rec}} \frac{\left(\ln(\hat{R}_y) - \ln\left(\bar{R}_y\right)\right)^2}{\sigma^2}$
$\left(\ln\left(\hat{R}_y\right) - \ln\left(\bar{R}_y\right)\right) \sim N\left(0, \sigma^2\right)$ or $\ln\left(\hat{R}_y\right) \sim N\left(\ln\left(\bar{R}_y\right), \sigma^2\right)$	$n_{rec}\frac{\ln(2\pi)}{2} + n_{rec}\ln(\sigma) + \frac{1}{2}\sum_{y=1}^{n_{rec}} \frac{\left(\ln(\hat{R}_y) - \ln\left(\bar{R}_y\right)\right)^2}{\sigma^2}$

Table 2
Life history traits, and fishery and survey characteristics used in simulations. Separate simulations were run using low (0.15) and high (0.8) fully selected fishing mortalities.

Age	Low *M*	High *M*	Maturity (%)	Weight (kg)	Flat fishery selectivity	Domed fishery selectivity	Survey selectivity
1	0.2	0.6	0	0.15	0.05	0.05	0.10
2	0.2	0.6	46	0.40	0.20	0.20	0.50
3	0.2	0.6	97	0.60	0.40	0.40	1.00
4	0.2	0.6	100	0.90	0.50	0.50	1.00
5	0.2	0.6	100	1.25	0.80	0.80	1.00
6	0.2	0.6	100	1.65	1.00	1.00	1.00
7	0.2	0.6	100	1.85	1.00	0.80	1.00
8+	0.2	0.6	100	2.20	1.00	0.50	1.00

whether normal or log-normal likelihoods are used. The difference between log-likelihoods when specifying:

$$\hat{R}_y \sim LN\left(\ln\left(\bar{R}_y\right), \sigma^2\right) \quad (1)$$

or

$$\ln\left(\hat{R}_y\right) \sim N\left(\ln\left(\bar{R}_y\right), \sigma^2\right) \quad (2)$$

is $\Sigma_{y=1}^{n_{rec}} \ln\left(\hat{R}_y\right)$, which appears in the log-likelihood for the log-normal distribution, but not the normal (Table 1). Both of these distributional assumptions are equally justified theoretically, and in typical maximum likelihood estimation each option produces identical parameter estimates because this extra summation term is a constant in the objective function. For example, using these distributions for (log-) relative abundance indices would estimate identical parameter values. However, when specifying recruitment to be lognormally distributed, as in Eq. (1), the annual recruitments $\hat{R}_y$ are estimated parameters rather than data, which may be problematic because the extra summation term is no longer a constant. More specifically, the model fit may improve (i.e., the penalized likelihood increased) by reducing the scale of the recruitment estimates despite signals from other data sources, and induce biased estimation of various population attributes. Ignoring the extra summation term in the lognormal distribution as a constant, however, is technically incorrect when Eq. (1) is assumed. Furthermore, the fact that these two distributions are both equally justified in theory, but may not perform equivalently in practice suggests that the topic of distributional assumptions for penalized maximum likelihood warrants evaluation in application to recruitment as well as any other non-normal penalties that might be used.

The objective of this manuscript was to review the potential for biased estimation of spawning stock biomass (*SSB*), fishing mortality (*F*), and recruitment caused by using the lognormal distribution to penalize annual recruitment. Using a simulation study with the alternative assumptions of Eqs. (1) and (2), we evaluated the effect of the summation term $\Sigma_{y=1}^{n_{rec}} \ln\left(\hat{R}_y\right)$ in the log-likelihood on the performance of a statistical catch-at-age (SCAA) model.

2. Methods

2.1. Overview

The SCAA model used for all simulations was the Age Structured Assessment Program, version 3.0.8 (ASAP; Legault and Restrepo, 1999; NOAA, 2012). This version of ASAP does not include a bias correction term for recruitment deviations (i.e., $e^{-\frac{\sigma^2}{2}}$), but sensitivity analysis using an annually varying adjustment described by Methot and Taylor (2011) and Methot and Wetzel (2013) suggested conclusions about the relative performance of the lognormal and normal penalties were robust to this omission, but improvements in bias near the terminal years could be achieved (Appendix A).

Simulations were used to estimate bias in parameter estimates by fitting the SCAA model to pseudo-datasets. Differences in bias were contrasted for normal and lognormal penalty assumptions for recruitment by fitting models with the alternative assumptions to the same pseudo-datasets (i.e., fits with and without the summation term highlighted above; Table 1). The simulation experiment was repeated for different values of *F*, natural mortality (*M*), and selectivity patterns (see below). For each simulation experiment, 100 pseudo-datasets each 40 years long were generated for use in the SCAA model.

2.2. Simulations

2.2.1. True underlying dynamics and pseudo-data generation

True population characteristics were based on a generic fish species with the general characteristics of groundfish in the northeast United States. Fish were approximately 50% mature at age-2 and 100% mature by age-4 (Table 2). Mean weights-at-age were time invariant and were the same for harvested and spawning fish (Table 2). Maturity and weights-at-age were constant among all simulations. We performed separate simulation experiments for multiple values of other population characteristics in a full factorial design for two levels of fully-selected *F* (time invariant), two levels of age- and time-invariant *M*, and for flat topped and domed fishery selectivity (Table 2). This study design resulted in separate simulation experiments for each of 16 combinations (2 *F* values × 2 *M* values × 2 selectivity shapes × 2 with normal or lognormal penalty). Numbers-at-age in the first year of the simulations equaled the deterministic equilibrium values associated