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Shear dispersion in a capillary tube with a porous wall

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ABSTRACT

An analytical expression is presented for the shear dispersion during solute transport in a coupled system comprised of a capillary tube and a porous medium. The dispersion coefficient is derived in a capillary tube with a porous wall by considering an accurate boundary condition, which is the continuity of concentration and mass flux, at the interface between the capillary tube and porous medium. A comparison of the obtained results with that in a non-coupled system identifies three regimes including: diffusion-dominated, transition, and advection-dominated. The results reveal that it is essential to include the exchange of solute between the capillary tube and porous medium in development of the shear dispersion coefficient for the last two regimes. The resulting equivalent transport equation revealed that due to mass transfer between the capillary tube and the porous medium, the dispersion coefficient is decreased while the effective velocity in the capillary tube increases. However, a larger effective advection term leads to faster breakthrough of a solute and enhances mass delivery to the porous medium as compared with the classical double-porosity model with a non-coupled dispersion coefficient. The obtained results also indicate that the finite porous medium gives faster breakthrough of a solute as compared with the infinite one. These results find applications in solute transport in porous capillaries and membranes.

1. Introduction

The subject of dispersion of a solute in conduits of various geometries has been extensively studied by many researchers from both theoretical and experimental aspects in the last decades. Taylor (1953) first published his work on the dispersion of solute transport in laminar flow in a capillary tube, followed by a more rigorous derivation of Aris (1956). After that, the study of Taylor (1953) was generalized gradually to incorporate the effects of geometries (Smith, 1983; Kessler and Hunt, 1994; Berkowitz and Zhou, 1996; Morrill et al., 2009; Wang et al., 2012; Ng and Zhou, 2012; Bouquain et al., 2012; Griffiths and Stone, 2012; Raoof and Hassanizadeh, 2013; Bolster et al., 2014), flow regimes (Sleep and Sykes, 1989; Sarkar and Jayaraman, 2004; Jansons, 2006; Bouquain et al., 2012; Raoof and Hassanizadeh, 2013; Bolster et al., 2014), chemical reactions (Aris, 1959; Smith, 1983; Purnama, 1988; Berkowitz and Zhou, 1996; Lee et al., 2008), and many other physical aspects. One of the important aspects is the condition at wall. Depending on the nature of the problem, different conditions can exist at the wall, including no-flux (Taylor, 1953; Aris, 1956; Ippolito et al., 1994; Berkowitz and Zhou, 1996; Bauget and Fourar, 2008; Wang et al., 2012; Griffiths and Stone, 2012), adsorbing (Aris, 1959; Smith, 1983; Purnama, 1988; Berkowitz and Zhou, 1996; Wels et al., 1997; Lee et al., 2008), permeable (Beard and Wu, 2009; Griffiths et al., 2013; Shaw et al., 2014; Aquino et al., 2015), and porous (Roubinet et al., 2012; Heße et al., 2013; Griffiths et al., 2013; Dejam et al., 2014a; Aquino et al., 2015).

The study of dispersion of a solute in flow through a cylindrical tube when it is subjected to an exchange of matter at porous wall is of great importance in a series of practical applications in the fields of geohydrological sciences with application to contaminant transport in underground water resources (Le Borgne et al., 2010; Bolster et al., 2011), petroleum engineering with application to dispersion in hydrocarbon reservoirs (Whitaker, 1967; Hassinger and Rosenberg, 1968;

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Arya et al., 1988; Wu et al., 2010; Mostaghimi et al., 2012; Dejam et al., 2014a), and chemical engineering with application to membrane separation (Gu et al., 2006; Jia and Liu, 2013; Berezhkovskii and Skvortsov, 2013).

Transport in all of the aforementioned systems is most often a coupled problem because of the usually porous nature of the medium surrounding the capillary tube where the porous medium and capillary tube communicate with each other. However, previous analytical studies on determination of dispersion coefficient in coupled systems have been traditionally based on the assumption of no communication between the capillary tube and porous medium or so-called non-coupled approach, in which a no-flux boundary condition is assumed at the tube wall.

Several approaches, such as the classical double-porosity model, have been used to model flow and transport in coupled systems analytically (Tang et al., 1981; Sudicky and Frind, 1982; Maloszewski and Zuber, 1985; Gerke and van Genuchten, 1996; West et al., 2004; Wu et al., 2010; Fomin et al., 2011; Joshi et al., 2012; Sharifi Haddad et al., 2012; Roubinet et al., 2012; Heße et al., 2013; Houseworth et al., 2013; Mahmoudzadeh et al., 2013). However, these analytical works used some approximations to determine an analytical expression for the dispersion coefficient during solute transport in a coupled system comprised of a capillary tube and a porous medium.

Usually the classical double-porosity model accompanied with a non-coupled dispersion coefficient is used to describe solute transport in coupled systems. The assumptions of 1) using a source/sink term in the governing equations of the classical double-porosity model for handling the interaction between capillary tube and porous medium, 2) applying a non-coupled dispersion coefficient, and 3) ignoring porous nature of the capillary tube wall may be the sources of important errors in predictions of the breakthrough curves in the capillary tube and mass storages in the porous medium. The major accomplishment of this study is determination of the dispersion coefficient through development of mathematical models for solute transport in a capillary tube with a porous wall by imposing an accurate boundary condition at the interface between the porous medium and capillary tube. The current work does not consider the assumptions used in the classical double-porosity model with a non-coupled dispersion coefficient.

Advective–dispersive transport in coupled systems is significant in many practical applications and has been widely studied. To the best of our knowledge, derivation of an analytical expression for the shear dispersion coefficient in a capillary tube with a porous wall as well as development of analytical solution for the averaged equation have not been thoroughly addressed in previous analytical works.

Aris (1959) applied his first discussion on the solute dispersion in a fluid flowing within a tube (Aris, 1956) to obtain a generalized dispersion coefficient in a coupled system comprised of two phases, including gas of concentration c_1 and liquid of concentration ($c_2 = \alpha c_1$), which flow in the annular space between two coaxial tubes of radii r_0 and r_2 with the interface r_1 ($0 \le r_0 < r_1 < r_2$). His generalized dispersion coefficient can be presented as $RD_1 + (1\text{-}R)D_2 + [R(11\text{-}16R + 6R^2)U_1^2 r_1^2/48D_1] + [R^2(1\text{-}R)U_1^2(r_2 - r_1)^2/3D_2] + [R^2(1\text{-}R)^2 s^2 U_1^2/2k\alpha r_1]$, where the first and second terms are the molecular diffusion coefficients in the gas and liquid phases, the third and fourth terms are Taylor dispersion coefficients in the gas and liquid phases, and the last term is a term due to the finite rate of partition between the gas and liquid phases. In this expression, U_1 and U_2 are the mean velocities of gas and liquid phases, D_1 and D_2 are the mean diffusion coefficients of gas and liquid phases, k is the rate of transfer across interface between the gas and liquid phases, α is the proportionality constant between gas and liquid concentrations, and s and R are parameters defined as $s = \sqrt{(r_1^2 - r_0^2) + \alpha(r_2^2 - r_1^2)}$ and $R = [U_1(r_1^2 - r_0^2) + U_2\alpha(r_2^2 - r_1^2)]/(U_1 - U_2)[(r_1^2 - r_0^2) + \alpha(r_2^2 - r_1^2)]$. He used his approach for distillation and chromatographic columns, which are special cases of Taylor dispersion. It is worth noting that Aris (1959) did not present an analytical solution of the coupled partial differential equations for his general case.

Later, Gill and Sankarasubramanian (1970) presented an exact method of analysis of unsteady convective diffusion systems without interphase mass transfer across boundaries. Subsequently, Sankarasubramanian and Gill (1973) developed a technique to handle problems with interphase transport, e.g. dispersion of a non-uniform initial distribution in time-variable isothermal laminar flow in a tube with a first-order rate process at the tube wall.

Thereafter, Gupta and Gupta (1972) developed an analytical formula for the dispersion of a solute in a liquid flowing between two parallel plates associated with an irreversible first-order reaction. They investigated the influences of both homogeneous and heterogeneous reactions on the dispersion under isothermal conditions. Gupta and Gupta (1972) concluded that by increasing the reaction rate constant, the effective Taylor diffusion coefficient decreases for the case of homogeneous reaction within the bulk liquid. They found the same result for the case of heterogeneous reaction at the catalytic walls when the wall catalytic parameter for fixed reaction rate constant corresponding to the bulk reaction is increased.

Next, Smith (1983) investigated the impact of an adsorbing boundary condition at the wall on shear dispersion in plane and pipe flows. He concluded that because the boundary tends to be the region of the lowest velocity and of the strongest shear, the remaining solute experiences on average an increased advection velocity and a reduced rate of shear dispersion.

Later, Purnama (1988) addressed the contaminant dispersion due to mass transfer between a flowing region and an adjacent stationary medium. He derived expressions for the longitudinal shear dispersion coefficient by assuming that the mass flux at the boundary depends linearly upon the concentration at earlier times. Purnama (1988) also presented the detailed results for the effects both of reactions and of retention at the bed upon contaminant dispersion in turbulent open-channel flow.

Subsequently, Sarkar and Jayaraman (2002) studied the role of the irreversible boundary reaction on the dispersion of solutes in annular flows. Their presented mathematical model brings out the dispersive transport following the injection of a solute in terms of the three effective transport coefficients, including the exchange, the convection, and the dispersion coefficients. They found that by increasing the value of the absorption parameter, the exchange and the convection coefficients are increased while the dispersion coefficient is reduced.

Thereafter, Ng (2006) developed an asymptotic analysis for the advection–diffusion transport of a solute in combined