



More general capillary pressure and relative permeability models from fractal geometry

Kewen Li

Stanford University, United States
Yangtze University, China

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ABSTRACT

More general capillary pressure and relative permeability models were derived theoretically from fractal modeling of a porous medium. It was found that the new capillary pressure model could be reduced to the frequently-used Brooks–Corey capillary pressure model and the Li–Horne imbibition model when the fractal dimension of a porous medium takes specific values. This also demonstrates that the Brooks–Corey model and the Li–Horne model have a further confirmed theoretical basis. Capillary pressure data measured using mercury intrusion technique were used to verify the model. The results demonstrated that the new capillary pressure model could represent the capillary pressure curves in those rocks with fractures or with great heterogeneity while the existing models cannot. The new relative permeability models can be reduced to the Brooks–Corey relative permeability model in a specific case. It has been proved theoretically that the relative permeability of each phase in a smooth fracture is only a linear function of its own saturation. Relative permeability data were calculated using the new models and the model results were compared with experimental data measured using a steady-state technique. The comparison demonstrated that the relative permeability models and experimental results were consistent with each other.

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1. Introduction

Capillary pressure and relative permeability are important parameters in hydrology, petroleum, geothermal reservoir engineering, and many other areas. There are two frequently-used methods to obtain relative permeability. One is experimental measurement and another is theoretical calculation from capillary pressure data. It may be difficult to measure relative permeability in some cases such as extremely low permeability rocks and special fluid systems with significant phase transformation and mass transfer between two phases as pressure changes (Li and Horne, 2001, 2004). Partially because of this, relative permeability is often calculated using capillary pressure. Purcell (1949) developed a method to calculate permeability using pore size distribution derived from mercury-injection capillary pressure curves. This method has been used to calculate multiphase relative permeabilities, as

reported by Gates and Leitz (1950). Later, Burdine (1953) introduced a tortuosity factor in the model. Corey (1954) and Brooks and Corey (1966) summarized the previous work and modified the method by representing capillary pressure curve as a power law function of the wetting-phase saturation. The Brooks and Corey relative permeability model has been used in many fields. These include vadose zone studies, subsurface remediation of nonaqueous phase liquids (Parker et al., 1987), and oil–water flow in reservoir rocks (Moulu et al., 1997). Wu and Pan (2003) derived a class of analytical solutions for the transient flow into unsaturated rock matrix using specially correlated, physically meaningful relative permeability and capillary functions.

There have been a number of existing capillary pressure models (Corey, 1954; Thomeer, 1960; Brooks and Corey, 1966; Van Genuchten, 1980; Skelt and Harrison, 1995; Huang et al., 1997; Lenormand, 1990; Lenhard and Oostrom, 1998; Jing and Van Wunnik, 1998; Li and Horne, 2001; Delshad et al., 2003). After reviewing the related literature and analyzing the

E-mail address: kewenli@stanford.edu.

mathematical expressions of capillary pressure curves, the author found that most of the capillary pressure models can be represented as follows:

$$S_w^* = [S_f + (aP_c)^n]^{-c} \quad (1)$$

Where S_f is a constant. P_c is the capillary pressure, and S_w^* is the normalized wetting-phase saturation, which could be expressed as follows in the drainage case:

$$S_w^* = \frac{S_w - S_{wr}}{1 - S_{nwi} - S_{wr}} \quad (2)$$

where S_w is the saturation of the wetting phase, S_{nwi} is the initial saturation of the nonwetting phase and S_{wr} is the residual saturation of the wetting phase.

Eq. (1) is reduced to the Van Genuchten model when S_f equals to 1. For the Van Genuchten model, a , c , and n are parameters to be determined. Note that S_{nwi} was equal to zero in the case of Van Genuchten (1980).

If S_f is equal to 0, Eq. (1) can be rearranged as:

$$S_w^* = (aP_c)^{-nc} \quad (3)$$

Therefore Eq. (1) is reduced to the frequently-used Brooks and Corey model in the case in which S_f equals to 0 (see Eq. (3)):

$$P_c = p_e (S_w^*)^{-1/\lambda} \quad (4)$$

where p_e is the entry capillary pressure of the rock sample, λ is a constant. The relationships among the parameters in Eqs. (3) and (4) are expressed as:

$$p_e = \frac{1}{a} \quad (5)$$

$$\lambda = nc. \quad (6)$$

Eq. (4) is reduced to the Corey model (see Eq. (7)) when the value of λ is equal to 2. Corey (1954) found that oil–gas capillary pressure curves could be expressed approximately using the following formula:

$$1/P_c^2 = CS_w^* \quad (7)$$

In Corey's case, S_{wr} was the residual oil saturation and S_{nwi} was equal to zero.

Thomeer (1960) proposed a relationship between capillary pressure and mercury saturation empirically:

$$P_c = p_e \left(\frac{S_{Hg}}{S_{Hg\infty}} \right)^{-\frac{1}{F_g}} \quad (8)$$

Where S_{Hg} is the mercury saturation, $S_{Hg\infty}$ is the mercury saturation at an infinite capillary pressure, and F_g is the pore geometrical factor.

Jing and Van Wunnik (1998) proposed a capillary pressure function to interpret the data of core-scale flow experiments. The function is expressed as follows:

$$P_c = p_c^0 \left[\left(\frac{d}{S_w - S_{wr}} \right)^n + a \right] \quad (9)$$

where p_c^0 is the capillary pressure scaling factor, d is a constant to define the curvature, n is the asymmetry shape factor, and a is a constant to control the value of the entry capillary pressure.

Among all the capillary pressure models described previously, the Brooks–Corey (1966) capillary pressure model has been used frequently for the consolidated porous media. In the case of unconsolidated porous media, the most frequently-used capillary pressure model is the Van Genuchten (1980) model.

The Brooks–Corey capillary pressure model is appropriate to the drainage case. In the imbibition case, Li and Horne (2001) proposed a capillary pressure model:

$$P_c = p_{max} (1 - S_w^*)^{-\frac{1}{\lambda}} \quad (10)$$

where p_{max} is the capillary pressure at the residual nonwetting-phase saturation.

Skelton and Harrison (1995) proposed an empirical model to describe the relationship between water saturation and reservoir height above the oil–water contact. The model is expressed as follows:

$$S_w = 1 - a \exp \left(\frac{-b_0}{P_c + d} \right)^c \quad (11)$$

where a , b_0 , c , and d are constants. Note that P_c is equal to the reservoir height above the oil–water contact in this case.

There have been other capillary pressure models under different conditions such as those proposed by Huang et al. (1997) and Lenormand (1990). One common feature of the capillary pressure models described previously is that all were proposed empirically. The parameters (for example, a , b_0 , c , and d in Eq. (11)) involved in these models do not have a physical significance. Some researchers (Tyler and Wheatcraft, 1990; Rieu and Sposito, 1991a,b; Perrier, et al., 1996; Li, 2004a) derived the empirical Brooks–Corey capillary pressure model theoretically from fractal modeling of a porous medium. This may also explain why the Brooks–Corey capillary pressure model works so well in many cases. However there have still been experimental data for which the Brooks–Corey (1966) capillary pressure model does not work (Li and Horne, 2003).

Li and Horne (2003) reported that the Brooks–Corey capillary pressure model could be used to represent the curves of the rock without fractures (for example, Berea sandstone) but not for rock samples with many fractures (for example, the rock from The Geysers geothermal field). In a previous paper (Li, 2004b), the author also reported that the Brooks–Corey (1966) capillary pressure model could work for some rock samples but not for others, even though the core samples were from the same reservoir.

Interestingly, Li and Horne (2003) found that fractal curves inferred from capillary pressure data were good straight lines with goodness of fitting over 0.9 for all the rock samples, both those with and those without fractures. The fractal curves represent the relationship between the number of pores and the radius of the pore throats. Later the author (Li, 2004a) found similar phenomena for the core samples from an oil reservoir.

These findings imply that a more general capillary pressure model may exist to represent both the rock in which the Brooks–Corey model works and the rock in which the Brooks–Corey model does not work. In this study, such a generalized capillary pressure model was derived theoretically from fractal modeling. New two-phase relative permeability models were

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