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Analytical solution to transport in three-dimensional heterogeneous well capture zones

P. Indelman, S.C. Lessoff [1], G. Dagan *

Department of Fluid Mechanics and Heat Transfer, Faculty of Engineering,
Tel Aviv University, Ramat Aviv, 69978 Tel Aviv, Israel

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Abstract

Solute transport is investigated in a heterogeneous aquifer for combined natural-gradient and well flows. The heterogeneity is associated with the spatially varying hydraulic conductivity $K(x, y, z)$, which is modelled as a log-normal stationary-random function. As such, the conductivity distribution is characterized by four parameters: the arithmetic mean K_A, the variance σ_Y^2 ($Y = \ln K$), the horizontal integral scale I of the axisymmetric log-conductivity autocorrelation and the anisotropy ratio $e = I_v / I$ (I_v is the vertical integral scale). The well fully penetrates an aquifer of constant thickness B and has given constant discharge QB, while the background aquifer flow is driven by an uniform mean head-gradient, $-J$. Therefore, for a medium of homogeneous conductivity K_A, the steady-state capture zone has a width $2L = Q/(K_A|J|)$ far from the well (herein the term capture zone is used to refer both to the zone from which water is captured by a pumping well and the zone that captures fluid from an injecting well).

The main aim is to determine the mean concentration as a function of time in fluid recovered by a pumping well or in a control volume of the aquifer that captures fluid from an injecting well. Relatively simple solutions to these complex problems are achieved by adopting a few assumptions: a thick aquifer $B \gg I_v$ of large horizontal extent (so that boundary effects may be neglected), weak heterogeneity $\sigma_Y^2 < 1$, a highly anisotropic formation $e < 0.2$ and neglect of pore-scale dispersion. Transport is analyzed to the first-order in σ_Y^2 in terms of the travel time of particles moving from or towards the well along the steady streamlines within the capture zone. Travel-time mean and variance to any point are computed by two quadratures for an exponential log-conductivity two-point covariance. Spreading is reflected by the variance value, which increases with σ_Y^2 and I/L.

For illustration, the procedure is applied to two particular cases. In the first one, a well continuously injects water at constant concentration. The mean concentration as function of time for different values of the controlling parameters σ_Y^2 and I/L is determined within control volumes surrounding the well or in

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* Corresponding author. Tel.: +972 3 640 8392; fax: +972 3 640 7334.
E-mail addresses: slessoff@eng.tau.ac.il (S.C. Lessoff), dagan@eng.tau.ac.il (G. Dagan).

[1] Tel.: +972 3 640 7232; fax: +972 3 640 7334.

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piezometers. In the second case, a solute plume, initially occupying a finite volume Ω_0, is drawn towards a pumping well. The expected solute recovery by the well as a function of time is determined in terms of the previous controlling parameters as well as the location and extent of Ω_0.

The methodology is tested against a full three-dimensional simulation of a multi-well forced-gradient flow field test ([Lemke, L., W.B. II, Abriola, L., Goovaerts, P., 2004. Matching solute breakthrough with deterministic and stochastic aquifer models. Ground Water 42], SGS simulations). Although the flow and transport conditions are more complex than the ones pertinent to capture zones in uniform background flow, it was found that after proper adaptation the methodology led to results for the breakthrough curve in good agreement with a full three-dimensional simulation of flow and transport.

Keywords: Groundwater hydrology; Stochastic processes; Groundwater transport; Random media; Wells

1. Introduction

Wells operating in aquifers under natural-gradient flow conditions are of common occurrence. For a pumping well, the capture zone reaches a steady shape after a sufficiently long time has elapsed. In the case of a homogeneous aquifer, an injecting well generates a capture zone that is of identical, but reversed, shape.

While the determination of capture zones and the flow field in a homogeneous aquifer is a simple matter, the problem is quite complex if hydraulic conductivity is spatially variable. This topic was the object of a few recent studies in which flow was modelled as two-dimensional, with transmissivity regarded as a random space function (e.g. Varljen and Shafer, 1991; Guadagnini and Franzetti, 1999; van Leeuwen et al., 1999; Feyen et al., 2003; Lu and Zhang, 2003; Stauffer et al., 2004). The main aim was to investigate the uncertainty of the capture-zone geometry as related to the randomness of transmissivity.

In the present study, we address the problem of solute transport for such a configuration. This is related to important applications like injection of fresh water into a contaminated aquifer or vice-versa (e.g. in Israel, during the winter, slightly saline water is imported from the Kinneret Lake and stored in the coastal aquifer). Conversely, extraction of contaminated plumes takes place in clean-up operations. The major mechanism of solute spreading is that of macrodispersion, which in turn is related to the hydraulic conductivity spatial variability. This is essentially a 3D process, related to presence of layers and lenses of variable conductivity that are present as a rule in sedimentary formations. The heterogeneity horizontal scales are of the order of meters, whereas the vertical ones are usually smaller by a factor of 10, as found for instance at Cape Cod (e.g. Hess et al., 1992) and at the Borden Site (e.g. Woodbury and Sudicky, 1991).

In contrast, two-dimensional modelling is appropriate at a regional scale, which is much greater than the aquifer depth B. Then, the property of interest is the depth-averaged transmissivity whose integral scale is expected to be larger than B. In fact, Hoeksema and Kitanidis (1985), in an exhaustive analysis of data for many aquifers in North America and Rubin (2003) (Table 2.1), have identified integral scales of the order of hundreds to tens of thousands meters. As a consequence, the zone of the order of the integral scale adjacent to the well is practically of constant transmissivity and, if measurements are made at the well location, as often happens, the two-dimensional problem becomes essentially deterministic. Hence, transmissivity spatial variability does not contribute to solute spreading in the well capture zone that is of interest in most applications. It is essential, therefore, to consider the impact of 3D heterogeneity upon solute spreading in this zone.