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On the calculation of eddy diffusivity in the shelf water from radium isotopes: High sensitivity to advection

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ABSTRACT

The concentrations of the radium isotopes have been used in previous studies to estimate eddy diffusivity on the continental shelf. These studies assume that the advective transport of the radium isotopes is negligible. A theoretical investigation using an analytic model with advection indicates, however, that the eddy diffusivity thus estimated is highly sensitive to the advection. It is shown that the error can be very large even for an advective velocity of the order of 1 mm/s–1 cm/s. The sensitivity increases with the increase of the half life of the isotope. For a 1 mm/s advective velocity, the estimated eddy diffusivity for the radium isotope with the shortest half life (i.e. ^{224}Ra) is almost doubled. In addition, we also conclude that (1) advection has more important effects on smaller values of eddy diffusivity; (2) the effect of advection increases rapidly as advection increases; and (3) offshore advection tends to increase the apparent eddy diffusivity and inshore advection tends to decrease the apparent eddy diffusivity, if the advection is ignored. Based on these facts, an improved model is presented to calculate both advection and eddy diffusivity using the activities of two isotopes, which yields consistent diffusivity and advection from different isotope pairs.

1. Introduction

Radium isotopes in seawater have been used as passive tracers of coastal waters to determine eddy diffusivity and submarine ground water discharge in coastal ocean waters (Moore, 2000, 2007, 2010). The radium isotopes have a diverse range of decay constants, which equal to 0.0608 day^{-1}, 0.1894 day^{-1}, 4.33×10^{-4} year^{-1}, and 0.12 year^{-1} for ^{223}Ra, ^{224}Ra, ^{226}Ra, and ^{228}Ra, respectively. These values of decay coefficient correspond to timescales (half life) of 11.4 days (^{223}Ra), 3.66 days (^{224}Ra), 1601 years (^{226}Ra), and 5.75 years (^{228}Ra), respectively. The first two are much shorter than the shelf water renewal timescale on the continental shelf of the South Atlantic Bight (SAB), which is on the order of 3 months (Atkinson et al., 1978), while the last two are much longer than that. In the study of Moore (2000), the cross shelf advection was assumed negligible and an analytic solution to the linear diffusion-radioactive decay model was obtained, which was fitted to the radium isotope data from the SAB through a log-linear regression to the activity of individual isotopes. It was believed that advection would make the cross shelf distribution of the isotopes nonlinear on a log scale. Since the observational data from the SAB continental shelf did satisfy the log-linear relation quite well, therefore the advection must have been negligible (Moore, 2000). However, it appears that the eddy diffusivities calculated with different isotopes from the same survey have very different values. Besides, how advection would affect the log-linear relation is unknown. These facts prompt us to examine the question under what conditions can the advection be neglected. A more specific question is: assuming that the source is on the coast and that its distribution along the shore is uniform, how sensitive is the eddy diffusivity (calculated from the diffusion-radioactive decay model) to cross shelf advection? To limit the complexity of the problem, we pose this question in a mathematical way. In other words, under ideal conditions (e.g. uniform distribution along the shore; source distributed on the coast only, etc), if we can establish a theoretical relation between the actual eddy diffusivity K_h and the apparent eddy diffusivity $\tilde{K}_h$ obtained by ignoring the advection, then different advection may correspond to different $\tilde{K}_h$ for a given K_h. The sensitivity study examines the rate of change of $\tilde{K}_h$ to the change of advection u. Mathematically, we need to quantify $\partial\tilde{K}_h(u,K_h)/\partial u$, with K_h as a parameter in this relation. Only around those values of K_h and u that result in "small" values of $\partial\tilde{K}_h(u,K_h)/\partial u$, can we use the diffusion-radioactive decay model, that ignores the advection, to reliably obtain the approximate values of the eddy diffusivity.

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Now the question becomes this: can we find the relation between $\tilde{K}_h$ and u, for a given K_h? The answer is yes because the diffusion-radioactive decay model can be extended to a broader application which includes the mean advection. In other words, if a constant advective velocity can be used to approximate the cross shelf mean advection, a similar analytic solution can be obtained. This will allow us to examine the effect of advection on the eddy diffusivity. In the following sections, we will first develop such an extended model and provide the solution. The sensitivity study will then be made, following which we will develop another model to calculate both advection and eddy diffusivity using two isotopes. Intuitively, using a combination of two isotopes to calculate the eddy diffusivity allows the use of more information and thus should present more reliable results, given that the model assumptions are justified. For the model to be valid, we assume that there is along-shelf uniformity and that the only source is on the coast.

2. Sensitivity analysis

In general, the diffusion–advection–decay process of a passive-tracer isotope in seawater can be described by the following equation

$$\partial A / \partial t + u \partial A / \partial x = K_h \partial^2 A / \partial x^2 - \lambda A \tag{1}$$

where A, t, u, x, K_h, and λ are concentration (or activity), time, cross shelf advective velocity, cross shelf distance, horizontal eddy diffusivity, and radioactive decay coefficient, respectively. Among the parameters, λ is a known constant, K_h and u are unknown but are assumed constant here for simplicity. For a steady state, $\partial A / \partial t = 0$, and we have

$$\partial^2 A / \partial x^2 - (u / K_h) \partial A / \partial x - \lambda A / K_h = 0 \tag{2}$$

It is important to note that here the advective velocity u should be considered as a temporal mean value which does not include the tidal signal. In other words, it should be obtained by averaging over a few tidal cycles to filter out tides. Assume that the values of A on the coast are known and equal to A_0 and it approaches to zero at the outer shelf. The boundary conditions for A are then:

$$A|_{x=0} = A_0, \quad A|_{x \to \infty} = 0 \tag{3}$$

The corresponding solution of Eq. (2) is then

$$A = A_0 e^{\frac{u - \sqrt{u^2 + 4\lambda K_h}}{2K_h} x} \tag{4}$$

Equivalently, the solution can be expressed as a log-linear format

$$\log A = \log A_0 + \frac{u - \sqrt{u^2 + 4\lambda K_h}}{2K_h} x \tag{5}$$

This shows that if the cross shelf advection (flow) is a constant, the concentration or activity A will still be log-linearly distributed across the shelf if the only source is at the coast. When $u = 0$, the solution becomes that of Moore (2000), i.e.

$$\log A = \log A_0 - \sqrt{\frac{\lambda}{K_h}} x \tag{6}$$

Therefore, Eq. (5) is a special case of Eq. (6). This indicates that even if the distributions of the radium isotope across the shelf are log-linear, the advection is not necessarily negligible. Since both Eqs. (5) and (6) are log-linear, both can be used to fit radium isotope data as described in Moore (2000). Assuming that there is a constant advective velocity across the shelf, the method of Moore (2000) is equivalent to the calculation of an "apparent" eddy diffusivity $\tilde{K}_h$ that is related to the "true" eddy diffusivity K_h and the advective velocity u by

$$-\sqrt{\lambda / \tilde{K}_h} = \left(u - \sqrt{u^2 + 4\lambda K_h} \right) \Big/ (2K_h) \tag{7}$$

from which the "apparent" eddy diffusivity can be expressed explicitly as

$$\tilde{K}_h = \frac{\lambda}{\left(\frac{u - \sqrt{u^2 + 4\lambda K_h}}{2K_h} \right)^2} \tag{8}$$

In the above equation, the apparent eddy diffusivity is dependent on the decay coefficient λ, advection u, and the actual eddy diffusivity K_h. When $u = 0$, the apparent eddy diffusivity is equal to the actual eddy diffusivity. If a small change in u does not result in a large change in $\tilde{K}_h$, then we may conclude that $\tilde{K}_h$ is not sensitive to the advection u and $\tilde{K}_h$ will be a good approximation of K_h. Our question is then: how sensitive is the apparent eddy diffusivity to the advective velocity? In other words, how close are the apparent and true eddy diffusivities under different cross shelf velocity values? To answer this question, we now discuss the relationship between K_h and $\tilde{K}_h$ under different cross shelf velocity values by examining (1) the above equation and (2) the rate of change of $\tilde{K}_h$ with advection u, i.e. $\partial \tilde{K}_h / \partial u$, for all the Radium isotopes. The rate of change of $\tilde{K}_h$ with respect to advection u can be shown to be

$$\frac{\partial \tilde{K}_h}{\partial u} = \frac{8\lambda K_h^2}{\left(u - \sqrt{u^2 + 4\lambda K_h} \right)^2 \cdot \sqrt{u^2 + 4\lambda K_h}} \tag{9}$$

While Eq. (9) provides a quantitative and formal sensitivity study, Eq. (8) provides some direct comparison between K_h and $\tilde{K}_h$. We will therefore first discuss Eq. (8) and then Eq. (9).

Figs. 1–4 show the results for ^{223}Ra (Figs. 1 and 3) and ^{224}Ra (Figs. 2 and 4) from Eq. (8). The results for ^{226}Ra and ^{228}Ra show much larger range of variations of $\tilde{K}_h$ such that it is difficult to make contour plots and the results will be discussed later. They are, however, of less interest because of their rather long half life timescales (much longer

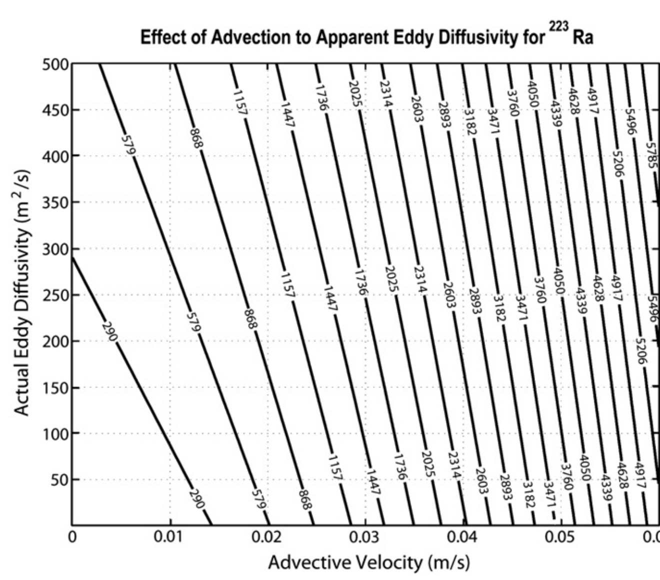

Fig. 1. Effect of offshore (positive) cross shelf advection on the apparent eddy diffusivity (shown by the contours) for ^{223}Ra, obtained from Eq. (8).