Ocean Modelling 107 (2016) 21–27

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

A diffusion approximation for ocean wave scatterings by randomly distributed ice floes

Xin Zhao[a,*], Hayley Shen[b]

[a] Advanced Research Institute for Multidisciplinary Science, Beijing Institute of Technology, Beijing, 100081, China
[b] Department of Civil and Environmental Engineering, Clarkson University, Potsdam, NY, 13699, USA

ARTICLE INFO

Article history:
Received 3 February 2016
Revised 20 September 2016
Accepted 24 September 2016
Available online 26 September 2016

Keywords:
Sea ice
Wave scattering
Diffusion approximation
Directional spreading

ABSTRACT

This study presents a continuum approach using a diffusion approximation method to solve the scattering of ocean waves by randomly distributed ice floes. In order to model both strong and weak scattering, the proposed method decomposes the wave action density function into two parts: the transmitted part and the scattered part. For a given wave direction, the transmitted part of the wave action density is defined as the part of wave action density in the same direction before the scattering; and the scattered part is a first order Fourier series approximation for the directional spreading caused by scattering. An additional approximation is also adopted for simplification, in which the net directional redistribution of wave action by a single scatterer is assumed to be the reflected wave action of a normally incident wave into a semi-infinite ice cover. Other required input includes the mean shear modulus, diameter and thickness of ice floes, and the ice concentration. The directional spreading of wave energy from the diffusion approximation is found to be in reasonable agreement with the previous solution using the Boltzmann equation. The diffusion model provides an alternative method to implement wave scattering into an operational wave model.

1. Introduction

Propagation of ocean waves into an ice cover is one of many wave phenomena in nature. It shares the same basic conceptual model as in acoustic, elastic, and electromagnetic wave propagation in complex media. The study of ocean waves in ice covered condition has a long history (e.g. Greenhill, 1886). Contemporary studies of this topic have been accelerating due to the rapid decline of ice in the Arctic (Comiso et al., 2008) and intensified wave activities (Thomson and Rogers, 2014). These conditions combined with increased shipping and environmental concerns call for better models of ocean waves in various ice covers.

As a material, ice covers are extremely inhomogeneous. Their physical properties change dynamically in response to both thermal and mechanical forcing. When ocean waves enter an ice cover, two things may happen: its speed may change and its energy may be reduced/redirected. Two fundamental processes affect the energy: the intrinsic and the scattering attenuation. The first results in net energy loss due to various dissipative processes, many of which have been considered in different models. The second is a reduction of energy in the original wave direction through scattering. The total energy is not affected but only redistributed into other directions. The present study addresses the scattering part of wave propagation.

Scattering is the directional redistribution from the original wave direction (Ishimaru, 1978). Studies at the scatterer scale are the foundation for the macro-scale models, which determine the energy propagation through a large collection of scatterers over a long distance. For instance, Ryzhik et al. (1996) gave the formulation of a general transport equation for wave propagation in random media including both the intrinsic and scattering attenuation. These attenuations from a single scatterer determine the coefficients used in the transport equation. Examples were provided in acoustic, electromagnetic, and elastic waves. Numerous references can be found in each of these fields as listed in their study.

For ocean waves under ice covers, the scattering process has also been developed from the scatterer scale to the macro-scale. At the scatterer scale, detailed study was conducted for 2-D wave transmission and reflection between open water and ice covers, between ice covers of different properties, where the ice cover was assumed semi-infinite or finite in extent, and for 3-D cases where the ice was circular or arbitrary in shape (reviewed in Squire, 2011). These studies provided the reflection from a single ice boundary in the 2-D case (e.g. Fox and Squire, 1990), and the scattering distribution from a single ice floe in the 3-D case

* Corresponding author.
E-mail address: xinzhao@clarkson.edu (X. Zhao).

http://dx.doi.org/10.1016/j.ocemod.2016.09.014

(e.g. Meylan and Squire, 1996). Utilizing these results, wave propagation through an array of ice floes in 2-D and 3-D cases, with uniform or non-uniform floe sizes and various ice properties have been studied (Doble and Bidlot, 2013; Dumont et al, 2011; Masson and LeBlond, 1989; Meylan and Masson, 2006; Meylan et al., 1997; Montiel et al., 2016; Perrie and Hu, 1996, 1997; Peter et al., 2004; Williams et al., 2013a, 2013b). The goal of these studies is to incorporate the scattering process in an operational ocean model.

There are two approaches used in solving the wave scattering through a large array of ice floes. One assumes the independence of phase interactions, so that the wave fields generated may be superimposed (e.g. Masson and Leblond, 1989; Meylan et al., 1997). The other solves the multiple-floe domain as a coupled hydroelastic problem with each floe-water interface as part of the complex boundary (Bennetts and Squire, 2009; Montiel et al., 2016). In the present study, we propose a different approach from these two. The proposed method is based on a modified diffusion approximation used in other wave propagation fields. This method is not as accurate as the two approaches mentioned above, but it provides an alternative method which may be easier to incorporate in operational ocean models that need to treat a large variety of ice covers.

2. The theoretical formulation

In this section, we derive the governing equations for wave scatterings with a diffusion approximation. The advantage of such approximation is to avoid calculating the complex integral kernel in the integral-differential equation of the wave action density function. The diffusion approximation is commonly used in the radiative transfer problem in a random medium (Ryzhik et al., 1996). However, the existing diffusion approximations used in various fields with random scatterers all assume strong scattering, such that the distance over which a single direction wave ray becomes isotropic is short compared with other length scales in a field of scatterers. This assumption allows previous diffusion models to focus on the isotropic part of the wave action density function.

For gravity waves propagating in a field of discrete ice floes, such assumption does not apply well to long waves. We thus propose here a different approach. The general philosophy of this approach is to start with a two-term decomposition for the wave action density function: the transmitted part and the scattered part. The transmitted part attenuates its energy through scatterings. The scattered part gains the energy from the transmitted part and gradually becomes more isotropic. We apply the diffusion approximation for the scattered part to obtain three differential equations. Details of the derivation are given below.

The wave action balance equation of ocean waves is

$$\frac{\partial}{\partial t}N(\mathbf{x},t,\mathbf{k}) + \nabla\cdot[\mathbf{c}_g N(\mathbf{x},t,\mathbf{k})] = \frac{S(\mathbf{x},t,\mathbf{k})}{\omega}. \tag{1}$$

Here, $N = E/\omega$ is the wave action density function (Andrews and Mcintyre, 1978), in which E is the wave energy density per unit area of angular frequency ω, $\mathbf{k}$ is the wave number vector, $\mathbf{x}$ is the spatial coordinate, t is time, $\mathbf{c}_g$ is the group velocity vector, and S is the total source/sink term. In addition to scattering, the source/sink may include processes such as wind generation, wave breaking, and nonlinear transfer between different frequencies. In the presence of an ice cover, these source/sink terms are not well established. If we ignore all other processes and focus on the scattering process alone, then along each wave component $\mathbf{k}$ the above equation becomes the following Boltzmann equation (Meylan et al., 1997).

$$\begin{aligned}\frac{\partial}{\partial t}N(\mathbf{x},t,k,\theta) &+ c_g\theta\cdot\nabla N(\mathbf{x},t,k,\theta)\\ &= -c_g\alpha_s(\mathbf{x},t,k,\theta)N(\mathbf{x},t,k,\theta)\\ &\quad + c_g\int_0^{2\pi} S_k(\mathbf{x},t,k,\theta,\theta')N(\mathbf{x},t,k,\theta')d\theta'.\end{aligned} \tag{2}$$

Here, α_s is the scattering attenuation coefficient, θ indicates the direction of wave number vector $\mathbf{k}$, $k=|\mathbf{k}|$, and S_k is the kernel function of wave energy redistribution. The scattering kernel S_k represents wave energy in the θ' direction that is redirected into the θ direction (Meylan et al., 1997). There has been a considerable amount of study based on thin-elastic-plate theory to derive S_k (e.g. Meylan and Squire, 1996; Meylan et al., 1997; and Bennetts and Williams, 2010), which we referred to as the "scatterer scale" studies. The energy conservation condition leads to

$$\alpha_s(\mathbf{x},t,k,\theta) = \int_0^{2\pi} S_k(\mathbf{x},t,k,\theta',\theta)d\theta'. \tag{3}$$

Thus, the redistribution of energy is exactly the loss of energy in the given wave direction.

We now propose a decomposition of N, which is more effective in following both weak and strong scattering processes. The wave action density function N defined in Eq. (1) is linearly decomposed into two parts,

$$N(\mathbf{x},t,k,\theta) = A(\mathbf{x},t,k,\theta) + B(\mathbf{x},t,k,\theta). \tag{4}$$

We define $A(\mathbf{x}, t, k, \theta)$ as the transmitted part in the direction of $N(\mathbf{x}, t, k, \theta)$, and $B(\mathbf{x}, t, k, \theta)$ is the scattered part. In this decomposition, the amount of the "incident" wave N that remains in the same direction after scattering is isolated from the rest of scattering energy. In this way, we can better treat weakly scattering waves before they become completely isotropic. Furthermore, the evolution of waves from open water, a no scattering region, into an ice field, a scattering region, may also be followed more closely near the boundary between the two regions. Masson and LeBlond (1989) applied a similar decomposition. The governing equations for these two parts are as the following,

$$\frac{\partial}{\partial t}A(\mathbf{x},t,k,\theta) + c_g\theta\cdot\nabla A(\mathbf{x},t,k,\theta) = -c_g\alpha_s(\mathbf{x},t,k,\theta)A(\mathbf{x},t,k,\theta), \tag{5}$$

$$\begin{aligned}\frac{\partial}{\partial t}B(\mathbf{x},t,k,\theta) + c_g\theta\cdot\nabla B(\mathbf{x},t,k,\theta) &= -c_g\alpha_s(\mathbf{x},t,k,\theta)B(\mathbf{x},t,k,\theta)\\ &+ c_g\int_0^{2\pi} S_k(\mathbf{x},t,k,\theta,\theta')B(\mathbf{x},t,k,\theta')d\theta'\\ &+ c_g\int_0^{2\pi} S_k(\mathbf{x},t,k,\theta,\theta')A(\mathbf{x},t,k,\theta')d\theta'.\end{aligned} \tag{6}$$

Eq. (5) says energy transmitted in the "incident" wave direction is reduced exactly by the amount of loss from the scattering process. Eq. (6) says the scattered energy is increased by contributions of the total redistribution from the scattered part and the "incident" part. The governing equation for A is straightforward. To simplify the integral-differential equation for B, we adopt a diffusion approximation.

To use the diffusion approximation, we decompose B into a directional averaged part and a fluctuating part as the following,

$$B(\mathbf{x},t,k,\theta) = \bar{B}(\mathbf{x},t,k) + B'(\mathbf{x},t,k,\theta) + \cdots, \tag{7}$$

where

$$\bar{B}(\mathbf{x},t,k) = \frac{1}{2\pi}\int_0^{2\pi} B(\mathbf{x},t,k,\theta)d\theta. \tag{8}$$