Ocean Modelling 107 (2016) 64–81

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

Linear and nonlinear properties of reduced two-layer models for non-hydrostatic free-surface flow

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ARTICLE INFO

Article history:
Received 21 February 2016
Revised 29 September 2016
Accepted 2 October 2016
Available online 4 October 2016

Keywords:
Non-hydrostatic model
Pressure Poisson equation
Two-layer flow
Dispersion
Shoaling gradient
Nonlinearity

ABSTRACT

A two-layer model with uniform non-hydrostatic pressure in the bottom produces favorable dispersion properties for coastal wave transformation at the computational requirements of a one-layer model. We derive the nonlinear governing equations and the corresponding dispersion relation, shoaling gradient, and super- and sub-harmonics to understand the theoretical performance of this reduced model. With the layer interface near the bottom, the dispersion relation shows an extended applicable range into deeper water at the expense of a slight overestimation of the celerity in intermediate water depth. The shoaling gradient rapidly converges to the exact solution in the shallow and intermediate depth range. These complementary characteristics allow identification of an optimal interface position for both linear wave properties. The resulting model exhibits good nonlinear performance in shallow and intermediate water depth and produces super- and sub-harmonics comparable to a two-layer model. Numerical tests involving standing waves show the reduced model has smaller discretization errors in the dispersion relation comparing to a one-layer model. Case studies of regular wave transformation over a submerged bar and a uniform slope provide comparison with laboratory data and demonstrate the linear and nonlinear properties derived from the governing equations. The good shoaling and nonlinear properties give rise to accurate waveforms in both cases, while dispersion errors from the governing equations and numerical schemes accumulate over time leading to phase shifts of the modeled waves.

1. Introduction

Ocean surface waves in the nearshore region drive a series of physical processes that shape the shoreline and impact the coastal community. Numerical modeling provides a cost-effective way to study these processes for coastal hazard assessment and engineering design. Among the various models, the non-hydrostatic approach has gained increasing attention from the research and engineering communities because of its favorable linear and nonlinear wave properties, robustness of the numerical scheme, and scalability with a finite number of layers to achieve desirable accuracy. The rapid convergence of the primary properties such as wave dispersion and flow kinematics demonstrates its capabilities for modeling of broad-spectrum ocean waves in practical applications (Bai and Cheung, 2015).

The methodology has undergone rapid development since Casulli (1995) introduced a non-hydrostatic pressure component in the Navier-Stokes equations for modeling of free-surface flow. The fractional step method of Casulli and Stelling (1998) solves the problem with a semi-implicit scheme and determines the non-hydrostatic pressure from a Poisson equation. Stelling and Zijlema (2003) proposed a Keller-box scheme with a boundary-fitted coordinate system to improve convergence of the dispersion relation. The use of conserved variables through layer-averaging can describe discontinuous flows involving hydraulic jumps or bores. The non-hydrostatic pressure in the resulting multi-layer model follows a piecewise linear distribution. Zijlema and Stelling (2005) utilized a pressure correction technique in a vertical boundary-fitted coordinate system to accurately compute the non-hydrostatic pressure field. Zijlema et al. (2011) implemented the approach with turbulence dissipation for modeling of coastal wave processes. Bai and Cheung (2013b) transformed the governing equations of a multi-layer model into a depth-integrated system, which enhances numerical stability under strong advection. The new system allows derivation of the dispersion relation and super- and sub-harmonics, which in turn provide an effective way to optimize the model performance through adjustment of the interface positions.

One-layer non-hydrostatic models are computationally efficient but cannot accurately resolve wave dispersion, especially in deep water. Two-layer models are able to cover an extended range of water-depth parameters crucial for describing coastal wave and

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http://dx.doi.org/10.1016/j.ocemod.2016.10.003

surf-zone processes. Adjustment of the interface position allows tuning of the model for a specific range of depth parameters. Nevertheless, the pressure Poisson equation remains the most computationally intensive part in the solution. Parameterization of the non-hydrostatic pressure profile can reduce computational costs, while retaining fundamental wave properties for practical application. Bai and Cheung (2013a) introduced a parameterized two-layer model with a prescribed ratio of the interface and bottom non-hydrostatic pressure. This allows elimination of the interface non-hydrostatic pressure from the Poisson equation and reduces the computational requirements to those of a one-layer model. An interface position at the mid flow depth gives rise to an optimal solution in terms of dispersion, shoaling, and nonlinear properties for the two-layer model (Bai and Cheung, 2012). Optimization of the pressure ratio yields a dispersion relation equivalent to a [2, 2] Padé approximation as with the Boussinesq model of Nwogu (1993).

The efficiency and scalability of the parameterized two-layer model enable studies of coastal wave processes and flood hazards on a regional scale. By adjusting both the interface position and pressure ratio, Cui et al. (2014) identified promising dispersion properties for nearshore wave transformation with a uniform pressure distribution in the bottom layer. A minor error exists in their dispersion relation, but does not affect its optimization through the interface position (see Appendix A). We provide an in-depth study of this unique pressure distribution through the nonlinear governing equations of the parameterized two-layer model and examine its optimization and performance in terms of the water depth parameter. In addition to the theoretical dispersion, we investigate effects of spatial discretization on the model celerity and derive the shoaling gradient and super- and sub-harmonics from the governing equations for comparison with the exact solutions from Airy and Stokes wave theories. A selection of numerical experiments involving wave dispersion, shoaling, and nonlinearity allows evaluation of the model capability in relation to conventional one- and two-layer models as well as laboratory data.

2. Governing equations

The properties of the parameterized two-layer system depend on the constructs of the governing equations that need elaboration and examination. We begin with layer-integration of the continuity and Euler equations and then convert the nonlinear governing equations into a depth-integrated system. Introduction of the interface position and pressure ratio as free parameters allows optimization of linear and nonlinear properties. The selection of the evolution valuable for the pressure profile is discussed and its influence on the non-hydrostatic character of the model is investigated.

2.1. Layer-integrated system

Consider a free-surface flow in a two-dimensional Cartesian coordinate system (x, z) as shown in Fig. 1. The surface elevation ζ, which is measured from the still-water level at $z = 0$, evolves with time t. The water depth is denoted by d and the flow depth by $h = \zeta + d$. We utilize the dispersion and nonlinear parameters μ and ϵ, which denote the depth to wavelength and amplitude to depth ratios, to nondimensionalize the continuity and Euler equations as

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + \epsilon\frac{\partial u^2}{\partial x} + \epsilon\frac{\partial uw}{\partial z} + \frac{\partial \zeta}{\partial x} + \mu^2\frac{\partial q}{\partial x} = 0 \tag{2}$$

$$\frac{\partial w}{\partial t} + \epsilon\frac{\partial uw}{\partial x} + \epsilon\frac{\partial w^2}{\partial z} + \frac{\partial q}{\partial z} = 0 \tag{3}$$

where (u, w) and q denote the velocity and non-hydrostatic pressure (Bai and Cheung, 2013b). The kinematic boundary conditions at the free surface and seabed become

$$w_\zeta = \frac{\partial \zeta}{\partial t} + \epsilon u_\zeta \frac{\partial \zeta}{\partial x} \qquad z = \epsilon\zeta \tag{4}$$

$$w_d = -u_d \frac{\partial d}{\partial x} \qquad z = -d \tag{5}$$

where u_ζ and u_d denote the horizontal velocity evaluated at the free surface and bottom respectively. The dynamic boundary condition defines the non-hydrostatic pressure $q_\zeta = 0$ at the free surface.

The flow depth is now defined as $h = \epsilon\zeta + d$ after nondimensionalization. The flow consists of two layers with the bottom and top thickness defined by $h_1 = \alpha h$ and $h_2 = (1-\alpha)h$ through an adjustable parameter $\alpha \in [0, 1]$. The position of the layer interface can be expressed in terms of the bottom layer thickness h_1, the top layer thickness h_2, or the surface elevation ζ as

$$\begin{aligned} z_\alpha &= h_1 - d \\ &= \epsilon\zeta - h_2 \\ &= \alpha\epsilon\zeta - (1-\alpha)d \end{aligned} \tag{6}$$

The governing equations for the two-layer flow can be derived from the continuity and Euler equations (1)–(3). Integration of the continuity equation (1) over the two layers in conjunction of the Leibniz rule yields

$$\frac{\partial h_1 u_1}{\partial x} - u_{z_\alpha}\frac{\partial z_\alpha}{\partial x} + w_{z_\alpha} = 0 \tag{7}$$

$$\frac{\partial h_2 u_2}{\partial x} - \epsilon u_\zeta \frac{\partial \zeta}{\partial x} + u_{z_\alpha}\frac{\partial z_\alpha}{\partial x} + w_\zeta - w_{z_\alpha} = 0 \tag{8}$$

where u_1 and u_2 represent the averaged horizontal velocity in the bottom and top layers and $(u_{z_\alpha}, w_{z_\alpha})$ denotes the velocity evaluated at the interface z_α. The horizontal component is expressed as the average from the two layers as $u_{z_\alpha} = (u_1 + u_2)/2$. The vertical component w_{z_α} should be distinguished from the vertical velocity of the interface. Their difference is defined by

$$\Delta w_{z_\alpha} = w_{z_\alpha} - \left(\frac{1}{\epsilon}\frac{\partial z_\alpha}{\partial t} + u_{z_\alpha}\frac{\partial z_\alpha}{\partial x}\right) \tag{9}$$

In obtaining the continuity equations (7) and (8), we approximate the kinematic boundary conditions (4) and (5) as

$$w_\zeta = \frac{\partial \zeta}{\partial t} + \epsilon u_2 \frac{\partial \zeta}{\partial x} \qquad z = \epsilon\zeta \tag{10}$$

$$w_d = -u_1 \frac{\partial d}{\partial x} \qquad z = -d \tag{11}$$

which utilize the layer-averaged horizontal velocity components instead of those at the free surface and the bottom to compute the vertical velocity.

Integration of the horizontal momentum equation (2) defines the flux and force balance in the bottom and top layers. After