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Towards improved numerical schemes of turbulent lateral dispersion

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ABSTRACT

This paper focuses on an alternative approach of lateral turbulent dispersion, proposed by Benoit Cushman-Roisin in 2008, that is based on a linear increase of the width of dispersing patches in a field of isotropic horizontal turbulence. In the open ocean, this Richardson-like dispersion regime is a well-observed feature on sub-mesoscale length scales from 10 to 100 km. In this work, we successfully validate and calibrate the new diffusion scheme using Lagrangian particles and Eulerian tracer in turbulent velocity fields simulated with the shallow-water equations. In discretized form, the new diffusion scheme exclusively relies on specification of a turbulent velocity scale that, unlike the turbulent diffusivity of Fickian approaches, is well defined through statistical properties of the turbulent flow.

1. Introduction

Diffusion is a fundamental physical process that modifies distributions of a property under the influence of molecular or turbulent velocity fluctuations. The accurate description of turbulent diffusion is important for many different scientific fields of study ranging from smaller-scale industrial engineering applications, pollutant dispersal (e.g. oil spills) modelling, the atmospheric sciences including weather forecasting, oceanography, the climate sciences including climate predictions, to studies of the magneto-hydrodynamics of the Earth's core. While this work focusses on turbulent diffusion in the ocean, its outcomes are also relevant to other disciplines.

The uncertainty inherent with the parameterization of sub-grid-scale processes, i.e. diffusion, in general circulation models (GCMs) of the ocean has been partially overcome over the last decades through advances in computer technology which made it possible to develop eddy-resolving hydrodynamic models (e.g. Semtner and Chervin, 1992). While such models are still restricted to simulation timescales of a few years in shallow shelf seas to decades in deep-sea applications, for efficiency reasons the ocean modules of climate forecasting models are still often based on a coarser grid resolution of 1º (such as climate models used for the 4th IPCC report) and thus particularly rely on the accuracy of turbulence closure models and their representation of diffusion. The accurate description of horizontal diffusion in eddy-resolving GCMs is similarly important as this modifies property distributions on unresolved lengths scales which, indeed, influence the resolved dynamics.

Horizontal diffusion in turbulent fluids is generally derived from Fick's laws of diffusion (Fick, 1855; Einstein, 1905), but with the use of an enlarged value of the constant of proportionality, D, called "eddy diffusivity" (e.g. Thorpe, 2007; Olbers et al., 2012). Under the assumption of a constant D, the related parabolic diffusion equation reads:

$$\frac{\partial C}{\partial t} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) \quad (1)$$

where C is the concentration of an Eulerian tracer, t is time, and x and y are horizontal Cartesian coordinates. When assuming constant eddy diffusivity, however, Fickian diffusion predicts that the length scale of a spreading patch, ℓ, increases with the square root of time (Cushman-Roisin, 2008); that is,

$$\ell^2 = 2Dt \quad (2)$$

where ℓ^2 is called "relative dispersion" (LaCasce, 2008; Koszalka et al., 2009). It has been argued for a long time, however, that the size of a dispersing patch in turbulent regime ought to grow faster than the square root of time (Batchelor and Townsend, 1956; Gifford, 1957), known as "super-diffusion", noting that Stommel (1949) concluded that the Fickian model fails to describe horizontal diffusion in the sea.

Observations in both water (e.g. Lawrence et al., 1995; Clark et al., 1996; Peeters and Hofmann, 2015) and air (e.g. Min et al., 2002) reveal patch sizes that grow roughly linearly with time. To reproduce this feature with the Fickian model, D in (2) has to be taken proportional to ℓ. This length-scale dependence of D is consistent with observational evidence (Fig. 1) derived from the dispersion of drifter clusters, where D is called "apparent" or "effective" diffusivity (e.g. Richardson, 1926; List et al., 1990). Consequently, the Fickian approach leads to a paradox situation in which it is impossible

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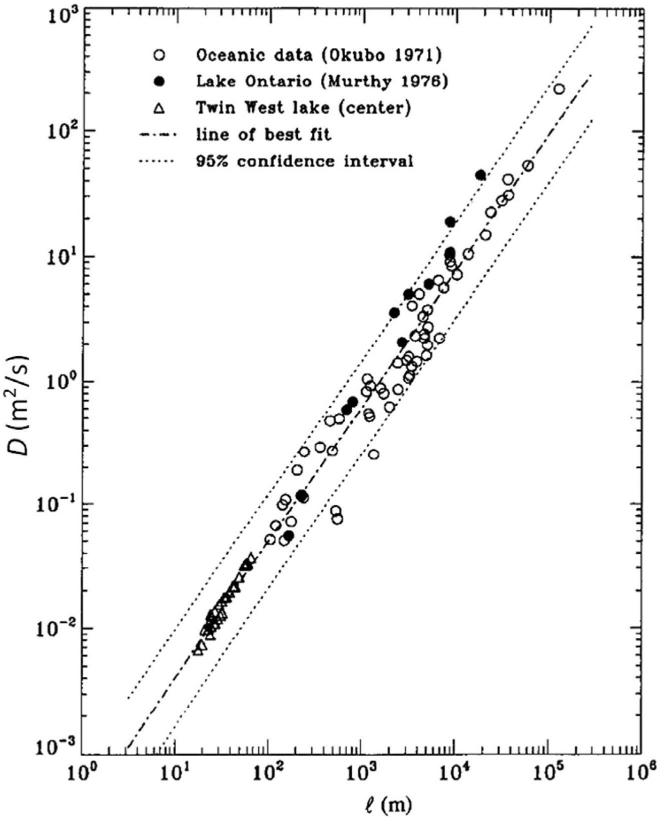

Fig. 1. Apparent diffusivity, D, vs. scale of diffusion, ℓ, for the central dye releases at Twin West Lake together with the data of Okubo (1974) and Murthy (1976). The line of best fit is given by $D = 3.2 \times 10^{-4} \ell^{1.1}$ (from Lawrence et al., 1995).

to tell which value of D to assign when two dispersing patches of different sizes merge.

The fact that Fickian diffusion does not perform well in describing the observed diffusive spreading is well known and documented. Super-diffusion appears to dominate the diffusive behaviour over many length scales under consideration. Super-diffusion occurs in the atmosphere (e.g., Richardson, 1926; Cushman et al., 2005), in geologic porous media (Cushman et al., 1994, Deng and Cushman, 1995, Meerschaert et al., 2001), lakes (e.g., Lawrence et al., 1995), vortex arrays in rotating flow (Weeks and Swinney, 1998), layered velocity fields (Zumofen et al., 1991), and eddy regimes of western boundary currents (Berti et al., 2011) to name a few examples. There is vast literature on the subject, the topic being carefully explored by the plasma community, which presents it as part of a more general problem (e.g., Isichenko, 1992; and references therein).

Different dispersion regimes have been identified in the open ocean. Based on 93 drifter pairs in the Nordic Seas that were initially separated less than 2 km from each other, Koszalka et al. (2009) concluded that relative dispersion in the open ocean exhibits three phases. The initial phase occurred during the first two days, at spatial scales less than 10 km, where dispersion increased approximately exponentially, $\ell^2 \sim \exp(t)$, with an e-folding time of roughly half a day. This exponential behaviour on small scales can be attributed to nonlocal stirring dominated by larger-scale eddies (Lin, 1972; Bennett, 1984; Babiano et al., 1990; LaCasce, 2008).

During the second, intermediate phase, from 2 to roughly 10 days and scales of 10 to roughly 100 km, the dispersion increased over time as a power law, $\ell^2 \sim t^R$, with an exponent of about 3 (Koszalka et al., 2009), known as Richardson law or Richardson–Obukhov law (Richardson, 1926; Obukhov, 1941; Cushman-Roisin, 2008). Note that the Richardson relation also corresponds to the case of an eddy diffusivity proportional to the 4/3 power of the length scale (Richardson and Stommel, 1948; Obukhov, 1941). Due to large uncertainties in the determination of eddy diffusivity from drifters (e.g. Klocker et al., 2012), this 4/3 power relation is generally statistically not distinguishable from a linear relationship, noting that a few years after Richardson's initial publication, Richardson and Gaunt (1930) revised the exponent downward by suggesting that the eddy diffusivity ought instead to grow as the first power of the patch size. Similarly, the analysis of 140 drifter pairs of the SCULP program in the Gulf of Mexico (Ohlmann and Niiler, 2005) revealed dispersion as $\ell^2 \sim t^R$ with an exponent of 2.2 rather than 3 (see Fig. 22b in LaCasce, 2008), which is close to linear spreading.

At larger spatial (> 100 km) and temporal (>2–3 weeks) scales, dispersion tends to increase linearly in time, $\ell^2 \sim t$, and pair velocities are uncorrelated (Koszalka et al., 2009; LaCasce, 2008; LaCasce et al., 2014), which is consistent with diffusive spreading according to (2). Hence, it appears that Fick's diffusion law applies only to oceanic processes on spatial scales > 100 km. This may justify the use of Fickian diffusion in non-eddy resolving climate models. In contrast, the intermediate oceanic regime on lengthscales of up 100 km appears to be controlled by super-diffusion. Hence, the use of the Fickian diffusion schemes on those lengthscales in eddy-resolving ocean GCMs is inconsistent with the observational evidence.

For clarity, it should be noted that stirring in the Richardson regime is still nonlocal in the sense that the tracer is partially stirred by eddies larger than the patch size, whereas in the Fickian regime the patch size outgrows the size of eddies so the stirring becomes local.

Cushman-Roisin (2008) proposed an alternative diffusion law that is based on the ensemble-average of the solution of the advection equation and the use of a probability density distribution for turbulent velocity fluctuations. Cushman-Roisin (2008) also demonstrates for the one-dimensional example that the new approach replicates a linear spreading of dispersing patches. For two-dimensional applications, this law is given by the integro-differential equation (Cushman-Roisin, 2013):

$$\frac{\partial C(x,y,t)}{\partial t} = A\langle u'\rangle \iint \frac{C(x',y',t) - C(x,y,t)}{\left[(x-x')^2 + (y-y')^2\right]^{3/2}} \mathrm{d}x'\mathrm{d}y' \qquad (3)$$

where A is an (unknown) numerical constant, and $\langle u'\rangle$ is a turbulent velocity scale. Furthermore, Cushman-Roisin and Jenkins (2006) show for one-dimensional momentum diffusion that the new diffusion operator conserves the total content of a property and that it dissipates the second moment (variance) of the property's distribution over time. It should also be highlighted that the discretized version of (3) can be written as an expansion series whereby each term is of the same form as the classical explicit Fickian scheme (see Section 3.2). Hence, the discretized version of (3) has the same general numerical characteristics as the Fickian scheme, except that includes more surrounding grid cells in the calculation.

The objective of this work is to validate and calibrate (3) on the basis of the analysis of the dispersion of both Lagrangian particles and Eulerian tracer in high-resolution hydrodynamic model applications that create fields of statistically well-defined turbulent velocity fluctuations.

2. Materials and methods

2.1. Description of hydrodynamic model

A simple shallow-water equation model (Kämpf, 2009) is applied to create a field of isotropic horizontal turbulence. To this