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Generation of long subharmonic internal waves by surface waves

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ABSTRACT

A new set of Boussinesq equations is derived to study the nonlinear interactions between long waves in a two-layer fluid. The fluid layers are assumed to be homogeneous, inviscid, incompressible, and immiscible. Based on the Boussinesq equations, an analytical model is developed using a second-order perturbation theory and applied to examine the transient evolution of a resonant triad composed of a surface wave and two oblique subharmonic internal waves. Wave damping due to weak viscosity in both layers is considered. The Boussinesq equations and the analytical model are verified. In contrast to previous studies which focus on short internal waves, we examine long waves and investigate some previously unexplored characteristics of this class of triad interaction. In viscous fluids, surface wave amplitudes must be larger than a threshold to overcome viscous damping and trigger internal waves. The dependency of this critical amplitude as well as the growth and damping rates of internal waves on important parameters in a two-fluid system, namely the directional angle of the internal waves, depth, density, and viscosity ratio of the fluid layers, and surface wave amplitude and frequency is investigated.

1. Introduction

The ocean is density-stratified due to vertical gradients in salinity, temperature, and sediment concentration. Stratification supports the generation of internal waves which are the main drivers of deep ocean mixing (Munk and Wunsch, 1998). Furthermore, observations in coastal waters indicate that breaking internal waves over a water-mud interface can result in increased water turbidity (Trowbridge and Traykovski, 2015). Internal waves have been studied widely in the past few decades, but many aspects of their generation, evolution and eventual dissipation still require further investigation (e.g. Garrett and Kunze (2007); Lamb (2014)).

The dynamics of wave processes in a stratified ocean can be illuminated by studying a fluid with a two-layer density structure. Koop and Butler (1981) showed that models based on Korteweg-de Vries (KdV) equations for weakly nonlinear long internal waves compare fairly well against experimental measurements where both fluid layers are shallow. KdV models are limited in that only unidirectional waves are treated. This limitation was removed in more recent models of Choi and Camassa (1996) and Lynett and Liu (2002) for weakly nonlinear and weakly dispersive waves, and in the fully nonlinear models of Choi and Camassa (1999) and Ostrovsky and Grue (2003). Debsarma et al. (2010) improved the (Choi and Camassa, 1999) model by including higher-order dispersive terms. The aforementioned models (except Choi and Camassa, 1996) impose rigid lid condition along the upper boundary, thus cannot capture surface-internal wave interactions. However, it is well-established that the coupling between surface and internal waves can modulate ocean wave spectra significantly (e.g. Watson et al. (1976); Olbers and Herterich (1979); Dysthe and Das (1981); Craig et al. (2011)).

One mechanism of internal wave generation is energy transfer from the surface to the interface through nonlinear interactions. Three classes of resonant generation of internal waves have been identified. Ball (1964) showed that an internal wave can grow due to energy transfer from two opposite-travelling surface waves in shallow water (class I). Segur (1980) noted the mathematical possibility of a triad formed by the interaction between a surface wave and two opposite-travelling oblique subharmonic internal waves (class II). This triad was studied experimentally by Foda and Huang (1994); Hill and Foda (1996); 1998); Jamali et al. (2003a), and recently by Fazeli et al. (2015). Furthermore, analytical studies were carried out to investigate the importance of different parameters in the two-fluid system on the growth of internal waves through this mechanism (Wen, 1995; Hill and Foda, 1996; Jamali et al., 2003a, b). These studies used a second-order perturbation theory to derive temporal evolution equations for the amplitude of the interacting waves. Hill (2004) applied a third-order theory and showed

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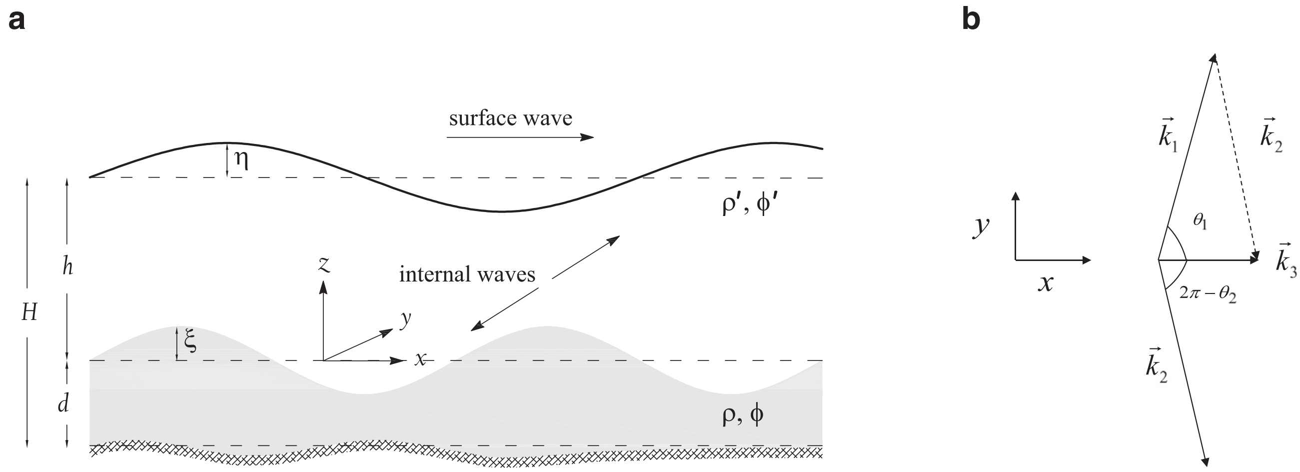

Fig. 1. (a) Definition sketch. (b) Direction of propagation of internal waves (1 and 2) and surface wave (3) in the horizontal plane.

that cubic nonlinear terms act to reduce internal wave growth rates, thus second-order theory overestimates their ultimate amplitudes. His study assumed a lower layer of infinite depth. Recently, Tahvildari and Jamali (2012) extended Hill's work to layers of arbitrary depth and added the effect of weak viscosity in the lower layer on internal wave growth. Alam (2012) showed that surface-internal wave coupling is possible in a co-propagating wave triad consisting of two relatively short surface waves and one internal wave with a much longer wavelength (class III).

In this study, we investigate the class II of resonant interaction with focus on long waves, in contrast to previous studies which mainly focused on short internal waves. A new set of two-layer Boussinesq equations is derived to study weakly nonlinear and weakly dispersive waves. This system extends the weakly dispersive limit of the Choi and Camassa (1996) equations to mildly varying bathymetry. Similar to the approach of Hill and Foda (1996) and Jamali et al. (2003a), a second-order perturbation method is applied to derive an analytical model for nonlinear interactions between one surface wave and two oblique internal waves. The analytical model is verified with the model of Jamali (1998) in the limit of shallow layers. The interaction coefficients are more straightforward to calculate compared to those in models based on the fully dispersive formulation (e.g. Jamali (1998)). The effect of weak viscosity in both fluid layers is added to the analytical model. While strong viscosity stratification can intensify interfacial disturbances (Harang et al., 2014), weak viscosity acts to inhibit interfacial wave growth (Davis and Acrivos, 1967). Jamali et al. (2003a) followed the approach of Davis and Acrivos (1967) and considered the effect of weak viscosity by adding a damping term to the evolution equations of wave amplitudes. Jamali et al. (2003a) use a damping coefficient that assumes identical viscosities in the fluid layers whereas we employ the formulation of Hill (2002) that accounts for varying viscosities and show that accounting for viscosity stratification is crucial in predicting internal wave growth. The analytical model is applied to investigate the effect of surface wave frequency and amplitude, and of the depth, density, and viscosity ratio of the fluid layers on the initial growth and damping rate of internal waves. As viscosity suppresses interfacial oscillations, the surface wave amplitude must exceed a critical value to overcome the damping effects and trigger internal waves. We use the analytical model to examine the effect of the aforementioned parameters on this critical amplitude.

The remainder of the paper is organized as follows. Section 2 summarizes the derivation of the two-layer Boussinesq equations. Section 3 outlines the development of an analytical model for surface-internal wave interactions using a perturbation approach. In Section 4, the dependency of the growth and damping rate of the internal waves and the critical surface wave amplitude on the important parameters in the system is examined. Conclusions and discussion are presented in Section 5.

2. Formulation

Fig. 1 (a) illustrates the configuration of the problem. The Cartesian coordinate system is introduced with origin at the undisturbed interface with the z-axis positive upward. The two-layer fluid is assumed to be laterally unbounded corresponding to an open ocean. The upper layer has density ρ' and depth h, and the lower layer has density ρ and spatially varying depth $d(x, y)$ (primed quantities refer to the upper layer). The total water depth is denoted by H. The free surface and the interface displacements are denoted by $\eta(x, y)$ and $\xi(x, y)$, respectively. The fluid layers are assumed inviscid, incompressible, homogeneous, immiscible, and the flow is assumed irrotational. Therefore, velocity potential functions, ϕ' and ϕ, can be introduced.

Mathematically, an infinite number of internal wave pairs can be in resonant interaction with a given surface wave. Each pair is characterized by its directional angle θ in the xy plane, where $\theta = 0$ corresponds to the surface wave direction and is measured positive counterclockwise. Previous theoretical and experimental studies (e.g. Hill and Foda (1996); Jamali et al. (2003a)) show that among all possible internal wave pairs, the one that includes identical counter-propagating waves that are subharmonic to the surface wave (with half the frequency of the surface wave) has the highest likelihood of occurrence. This wave triad is shown in Fig. 1(b) and will be the focus of our study.

2.1. Governing equations

The approach we take to deriving the Boussinesq equations is to start from the boundary value problem for potential flow. The internal kinematics in each layer are governed by the Laplace equation and scaled in a manner consistent with weakly dispersive waves in shallow water (e.g. see Mei et al. (2005)). The following dimensionless variables are introduced:

$$(\tilde{x}, \tilde{y}) = k_c(x, y), \quad \tilde{z} = \frac{z}{H_c}, \quad \tilde{t} = k_c\sqrt{gH_c}\, t, \quad (\tilde{\xi}, \tilde{\eta}) = \frac{(\xi, \eta)}{A},$$

$$(\tilde{d}, \tilde{h}) = \frac{(d, h)}{H_c}, \quad \tilde{\phi} = \phi\frac{k_c}{\epsilon\sqrt{gH_c}}, \quad \tilde{k} = \frac{k}{k_c}, \tag{1}$$

where the overtildes denote dimensionless quantities, A is a typical surface wave amplitude, k_c is a characteristic internal wavenumber,