Ocean Modelling 106 (2016) 121–130

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

NHWAVE: Consistent boundary conditions and turbulence modeling

Morteza Derakhti[a,*], James T. Kirby[a], Fengyan Shi[a], Gangfeng Ma[b]

[a] Center for Applied Coastal Research, University of Delaware, Newark, DE, USA
[b] Department of Civil and Environmental Engineering, Old Dominion University, Norfolk, VA, USA

ARTICLE INFO

Article history:
Received 1 June 2015
Revised 29 July 2016
Accepted 8 September 2016
Available online 12 September 2016

Keywords:
Non-hydrostatic model
Dynamic boundary conditions
Wave breaking

ABSTRACT

Large-scale σ-coordinate ocean circulation models neglect the horizontal variation of σ in the calculation of stress terms and boundary conditions. Following this practice, the effects of surface and bottom slopes in the dynamic surface and bottom boundary conditions have been usually neglected in the available non-hydrostatic wave-resolving models using a terrain-following grid. In this paper, we derive consistent surface and bottom boundary conditions for the normal and tangential stress fields as well as a Neumann-type boundary condition for scalar fluxes. Further, we examine the role of surface slopes in the predicted near-surface velocity and turbulence fields in surface gravity waves. By comparing the predicted velocity field in a deep-water standing wave in a closed basin, we show that the consistent boundary conditions do not generate unphysical vorticity at the free surface, in contrast to commonly used, simplified stress boundary conditions developed by ignoring all contributions except vertical shear in the transformation of stress terms. In addition, it is shown that the consistent boundary conditions significantly improve predicted wave shape, velocity and turbulence fields in regular surf zone breaking waves, compared with the simplified case. A more extensive model-data comparison of various breaking wave properties in different types of surface breaking waves is presented in companion papers (Derakhti et al., 2016a,b).

1. Introduction

Surface wave breaking plays an important role in numerous environmental and technical processes such as air-sea interaction, acoustic underwater communications, optical properties of the water column, nearshore mixing and coastal morphodynamics. Wave breaking is a highly dissipative process, limiting the maximum height of surface waves. It is also a source of turbulence, enhancing transport and mixing in the ocean surface layer (Banner and Peregrine, 1993; Melville, 1996; Duncan, 2001; Perlin et al., 2013).

Although large-eddy simulations (LES) combined with the volume-of-fluid (VOF) method for free-surface tracking (Watanabe et al., 2005; Derakhti and Kirby, 2014; 2016) can resolve turbulence and mean flow dynamics in breaking waves quite well, they are computationally expensive even for laboratory-scale events. A lower-resolution framework is needed to study long-term, $\mathcal{O}$(days), and large-scale, $\mathcal{O}(100\text{ m} \sim 10\text{km})$, wave-breaking-driven circulation as well as transport of sediment, bubbles, and other suspended materials. Computationally efficient Boussinesq-type models (e.g., Wei et al., 1995; Shi et al., 2012) can often yield acceptable predictions of surface elevations and depth-averaged currents in the nearshore region. Such single layer models, however, cannot provide the vertical structure of mean flow or information on instantaneous motion over rapidly-varying bathymetry or current fields, and thus recourse must be made to models which either provide estimates of vertical structures through closure hypotheses (Kim et al., 2009) or which utilize a three-dimensional (3D) framework from the outset.

During the past two decades, several multi-layered wave-resolving non-hydrostatic models based on Reynolds-averaged Navier–Stokes (RANS) equations, such as Stansby and Zhou (1998), Lin and Li (2002), Bradford (2011) and Ma et al. (2012), have been developed for coastal applications using surface- and terrain-following curvilinear (x, y, σ) coordinates, hereafter referred as the σ-coordinate system. In comparison with VOF-based models, a direct simplification of this new framework is achieved by assuming the free surface to be a single-valued function of horizontal location. By using a σ-coordinate system, the free surface is always located at an upper computational boundary, determined by applying free-surface boundary conditions. Using a Keller-box scheme, a pressure boundary condition at the free surface can thus be accurately prescribed, and dispersion characteristics of short waves are typically predicted accurately using a few vertical σ-levels.

* Corresponding author.
E-mail address: derakhti@udel.edu (M. Derakhti).

http://dx.doi.org/10.1016/j.ocemod.2016.09.002

However, the effects of surface and bottom slopes in the dynamic boundary conditions at the top and bottom interfaces, e.g., the continuity of the tangential surface stress, have been ignored in most of the previous non-hydrostatic studies using a terrain-following grid, following the previous practice in large-scale ocean circulation models. In the absence of surface wind stress, the simplified tangential stress boundary condition at the free surface developed by ignoring all contributions except vertical shear in the transformation of tangential stress, reads as (Lin and Li (2002, equation 41), Bradford (2011, equation 22) and Ma et al. (2012, equation 36))

$$\frac{\partial u_i}{\partial z} = \frac{1}{D}\frac{\partial u_i}{\partial \sigma} = 0, \tag{1}$$

where $i = 1, 2$ and u_i is the horizontal velocity component in the i direction. In intermediate and deep water, we have

$$\omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = \frac{u_0}{L_0}\left(\frac{\partial u'}{\partial z'} - \frac{\partial w'}{\partial x'}\right) \sim \frac{\varepsilon c_0}{L_0} \sim \varepsilon f_0 \tag{2}$$

where ω_y is the vorticity component in the y direction, L_0 and u_0 are the length and velocity scales respectively, for both the horizontal and vertical directions. Here, c_0 and f_0 are the wave's phase speed and frequency respectively, $\varepsilon = a_0/L_0$ is the wave steepness, and a_0 is the wave amplitude. Thus, imposing (1) generates error in wave vorticity divided by wave frequency to $\mathcal{O}(\epsilon)$, the order of the motion itself. In other words, imposing (1) at the free surface acts as an unphysical local source of vorticity of strength $\partial w/\partial x$ which in turn generates an unphysical near-surface residual circulation.

In breaking waves, the surface slopes are large, $\mathcal{O}(1)$, and $\partial u_i/\partial\sigma \neq 0$ in a bore-like region, and thus using (1) provides a poor estimation of the associated near-surface velocity gradient and turbulence production. Another simplification in some of the existing non-hydrostatic RANS models using the σ-coordinate system is the neglect of the effects of surface and bottom slopes in the horizontal diffusion terms (Stansby and Zhou, 1998).

Our goals here are (1) to derive consistent surface and bottom dynamic boundary conditions for the normal and tangential stress fields and (2) to carefully examine the role of surface slopes in the predicted near-surface velocity and turbulence fields in surface gravity waves. We compare the velocity field in a deep-water standing wave in a closed basin predicted by the new version of the non-hydrostatic model NHWAVE with that predicted by a previous version of the model (Ma et al., 2012) (hereafter referred to as the original model), showing that the consistent boundary conditions do not generate unphysical vorticity at the free surface, in contrast to commonly used, simplified stress boundary conditions developed by ignoring all contributions except vertical shear in the transformation of stress terms. In addition, it is shown that the consistent boundary conditions significantly improve the predicted wave shape and wave heights as well as velocity and turbulence fields in regular surf zone breaking waves, compared with the simplified case.

The paper is organized as follows. In Section 2, we present the governing equations, in conservative form, describing a complete form of the RANS equations in the σ-coordinate system together with various turbulence closure models. In Section 3, we derive the consistent surface and bottom dynamic boundary conditions for the velocity and dynamic pressure fields, using the appropriate dynamic boundary conditions on normal and tangential stresses at the top and bottom interfaces as well as a Neumann-type boundary condition for scalar fluxes. In Section 4, we examine the role of surface slopes in the near-surface velocity and turbulence fields in surface gravity waves. Wave-breaking-induced eddy viscosity and its effect on the wave height distribution in the surf zone are discussed in Section 5. Conclusions are given in Section 6. A more extensive model-data comparison of various breaking wave properties in different types of surface breaking waves is presented in companion papers (Derakhti et al., 2016a,b).

2. Governing equations in conservative form

Here, the complete and conservative form of the RANS equations and the scalar transport equation in the σ-coordinate system are presented. Further, different turbulence models including the standard $k-\epsilon$ (Rodi, 1980) and the renormalization group (RNG) approach by Yakhot et al. (1992), are presented. The surface and bottom boundary conditions will be derived in the next section. Details of the numerical method may be found in Ma et al. (2012) and Derakhti et al. (2015).

2.1. Continuity and momentum equations

Assuming a uniform density field, the RANS equations in Cartesian coordinates (x_1^*, x_2^*, x_3^*), where $x_1^* = x^*, x_2^* = y^*$ and $x_3^* = z^*$ reads as

$$\frac{\partial u_j}{\partial x_j^*} = 0 \tag{3}$$

$$\frac{\partial u_i}{\partial t^*} + \frac{\partial u_i u_j}{\partial x_j^*} = \frac{1}{\rho_0}\frac{\partial \mathscr{S}_{ij}}{\partial x_j^*} + g_i\delta_{i3}, \tag{4}$$

where $(i, j) = 1, 2, 3$, u is the ensemble-averaged velocity, ρ_0 is the reference water density, $\mathbf{g} = (0, 0, -g)$ is the gravitational acceleration, δ is the Kronecker delta function, $\mathscr{S}_{ij} = \Pi_{ij} - \tau_{ij}$ is the total ensemble-averaged stress tensor, Π_{ij} is the ensemble-averaged fluid stress and τ_{ij} is the Reynolds stress. For an incompressible fluid, the net ensemble-averaged fluid stress, composed of the pressure contribution p plus the viscous stress σ_{ij}, is defined by $\Pi_{ij} = -p\delta_{ij} + \sigma_{ij}$. In a Newtonian fluid, we may assume that $\sigma_{ij} = 2\mu e_{ij}$, where $e_{ij} = 1/2(\partial u_i/\partial x_j^* + \partial u_j/\partial x_i^*)$ is the strain rate tensor and μ is the dynamic viscosity. Although there is no universal model for τ_{ij}, even in the case of a single-phase flow, we use the common eddy viscosity approach to relate the anisotropic part of the Reynolds stress, τ_{ij}^{dev}, to the rate of strain, e_{ij} as $\tau_{ij}^{dev} \equiv \tau_{ij} - \frac{\delta_{ij}}{3}\tau_{kk} = -2\rho_0(\nu_t)_j e_{ij}$. Here, $(\nu_t)_j$ is the turbulent eddy viscosity in the j direction (j is not a free index here), obtained from an appropriate turbulence model. If grid resolution in the horizontal directions is considerably different from that in the vertical direction, the horizontal turbulent eddy viscosity $(\nu_t)_x = (\nu_t)_y$ may be different from that in the vertical direction $(\nu_t)_z$.

The governing Eqs. (3) and (4) are next transformed into the σ-coordinate system (see Fig. 1), which is given by

$$t = t^* \quad x = x^* \quad y = y^* \quad \sigma = \frac{z^* + h}{D} \tag{5}$$

where $D = h + \eta$ is the total water depth, h is the still water depth, and η is a free surface elevation. In the case of a multi-valued surface, however, the definition of a free surface elevation is arbitrary, and, we assume η is sufficiently smooth to be considered as a single-valued mean air-water interface.

Each term of (3) and (4) is transformed into the σ-coordinate system by multiplying by D and using chain differentiation rule as

$$\begin{aligned} D\frac{\partial \psi}{\partial x_j^*} &= D\frac{\partial \psi}{\partial x_j}\lambda_j + D\frac{\partial \psi}{\partial \sigma}\sigma_{x_j^*} \\ &= \frac{\partial D\psi}{\partial x_j}\lambda_j + \left(-\frac{D_{x_j}}{D}\lambda_j\right)D\psi + \left(\sigma_{x_j^*}\right)\frac{\partial D\psi}{\partial \sigma} \\ &= \frac{\partial D\psi}{\partial x_j}\lambda_j + \frac{\partial \sigma_{x_j^*} D\psi}{\partial \sigma}, \end{aligned} \tag{6}$$