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The adaptive EVP method for solving the sea ice momentum equation

Madlen Kimmritz[a,c,*], Sergey Danilov[a,b], Martin Losch[a]

[a] Alfred Wegener Institute, Bussestrasse 24, D-27570 Bremerhaven
[b] A. M. Obukhov Institute of Atmospheric Physics RAS, Moscow, Russia
[c] Nansen Environmental and Remote Sensing Center and Bjerknes Centre for Climate Research, Bergen, Norway

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ABSTRACT

Stability and convergence of the modified EVP implementation of the visco-plastic sea ice rheology by Bouillon et al., Ocean Modell., 2013, is analyzed on B- and C-grids. It is shown that the implementation on a B-grid is less restrictive with respect to stability requirements than on a C-grid. On C-grids convergence is sensitive to the discretization of the viscosities. We suggest to adaptively vary the parameters of pseudotime subcycling of the modified EVP scheme in time and space to satisfy local stability constraints. This new approach generally improves the convergence of the modified EVP scheme and hence its numerical efficiency. The performance of the new "adaptive EVP" approach is illustrated in a series of experiments with the sea ice component of the MIT general circulation model (MITgcm) that is formulated on a C-grid.

1. Introduction

The viscous-plastic (VP) rheology (Hibler III, 1979), connecting sea ice deformation rates with ice stresses, forms the basis of most climate sea-ice models. The resulting set of equations of ice dynamics is very stiff and thus calls for the design of efficient solution methods to avoid the restriction to very small time steps in standard explicit methods. Partial linearization allows the stiff part of the problem to be treated implicitly, but requires iterative solvers (Zhang and Hibler, 1997). Although this linearization lifts the time step restriction, it requires many (Picard) iterations to recover the full nonlinear solution. Traditionally only a few Picard iterations are made and convergence is sacrificed (Lemieux and Tremblay, 2009). This motivated the development of fully nonlinear Jacobian-free Newton-Krylov (JFNK) solvers (Lemieux et al., 2010, 2012; Losch et al., 2014). They converge faster than previous methods but still remain an expensive solution.

The elastic-viscous-plastic (EVP) method is an alternative to implicit methods. It relaxes the time step limitation of the explicit VP method by introducing an additional (artificial, not physically motivated) elastic term to the stress equations. This allows a fully explicit time stepping scheme with much larger time steps than possible for the VP method (Hunke and Dukowicz, 1997; Hunke, 2001), but still requires subcycling within the external time step commonly set by the ocean model. The effects of the additional elasticity term, however, are reported to lead to noticeable differences in the deformation field, and result in solutions with smaller viscosities and weaker ice (e.g., Lemieux et al., 2012; Losch et al., 2010; Losch and Danilov, 2012; Bouillon et al., 2013).

In many cases, these effects are linked to the violation of local stability limits (analogous to the Courant number constraint for advection) associated with the explicit time stepping scheme of the subcycling process (Hunke and Dukowicz, 1997; Hunke, 2001). Their most frequent manifestation is grid-scale noise in the ice velocity derivatives and hence in ice viscosities, in particular, on meshes with fine or variable resolution (Losch and Danilov, 2012) (the numerical code may remain stable and simulate smooth fields of ice concentration and thickness). In an attempt to improve the performance of the EVP method, a modification of the time-discrete model was proposed by adding an inertial time stepping term to the momentum balance (Lemieux et al., 2012). This mEVP (modified EVP) method was reformulated by Bouillon et al. (2013) as a "pseudotime" iterative scheme. By construction, it should lead to solutions that are identical to those of the VP method provided the scheme is stable and runs to convergence. The analysis of mEVP for a simplified one-dimensional (1D) case suggests that the stability is defined by a single parameter that depends on the resolution, the time step, the ice viscosity, and on the relaxation parameters of the pseudotime stepping (Bouillon et al., 2013; Kimmritz et al., 2015).

* Corresponding author. Tel.: +4917634440955.
E-mail addresses: madlen.kimmritz@awi.de (M. Kimmritz), sergey.danilov@awi.de (S. Danilov), martin.losch@awi.de (M. Losch).

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Although the 1D analysis is expected to be valid at least qualitatively in two dimensions (2D), there are a few aspects that are not covered by the 1D analysis: the velocity and stress divergence vectors are not collinear in 2D; velocities are staggered in space (on a C-grid) but are collocated on a B-grid, so that on a C-grid one works with normal velocity components rather than the full velocity vector (as on the B-grid); on C-grids the components of the strain rate tensor and the stress components are not collocated. These aspects affect the convergence properties of the method. Several C-grid implementations have been suggested in literature (e.g. Bouillon et al., 2013; Lemieux et al., 2012; Losch et al., 2010).

This work extends the analysis of Kimmritz et al. (2015) by exploring the impact of space discretizations on the stability properties of the mEVP method. Motivated by this analysis we propose a new adaptive EVP implementation (aEVP). In this scheme the parameters of the pseudotime stepping are locally adjusted in each pseudotime subcycle in order to ensure stability. In simple experiments we demonstrate that this scheme leads to a significant improvement of the convergence properties.

The article is organized as follows: In Section 2 we briefly review the governing equations, the mEVP scheme as formulated in Bouillon et al. (2013) and its discretization on B- and C-grids. We continue with the stability analysis of the linearized 2D equations in Section 3, and introduce the aEVP method and explore its stability properties in Section 5. In Section 6, we illustrate our results in experiments performed with the sea ice component of an ocean general circulation model (MITgcm, see the source code at http://mitgcm.org). Conclusions and outlook are given in Section 6.

2. Model description

The horizontal momentum balance of sea ice is written as

$$m(\partial_t + f\mathbf{k}\times)\mathbf{u} = a\boldsymbol{\tau} - C_d a\rho_o(\mathbf{u}-\mathbf{u}_o)|\mathbf{u}-\mathbf{u}_o| + \mathbf{F} - mg\nabla H. \quad (1)$$

Here m is the ice (plus snow) mass per unit area, f is the Coriolis parameter and $\mathbf{k}$ the vertical unit vector, a the ice concentration, $\mathbf{u}$ and $\mathbf{u}_o$ the ice and ocean velocities, ρ_o is the ocean water density, $\boldsymbol{\tau}$ the wind stress, H the sea surface elevation, g the acceleration due to gravity and $F_l = \partial\sigma_{kl}/\partial x_k$ the divergence of the internal stress tensor σ_{kl} (with indices k, l denoting x_1 and x_2 directions). We follow Bouillon et al. (2013) in writing the VP constitutive law as

$$\sigma_{kl}(\mathbf{u}) = \frac{P}{2(\Delta+\Delta_{min})}\left[(\dot\epsilon_d - \Delta)\delta_{kl} + \frac{1}{e^2}(2\dot\epsilon_{kl} - \dot\epsilon_d\delta_{kl})\right], \quad (2)$$

with

$$\dot\epsilon_{kl} = \frac{1}{2}(\partial_k u_l + \partial_l u_k), \quad \Delta = \left(\dot\epsilon_d^2 + \frac{1}{e^2}\dot\epsilon_s^2\right)^{1/2}. \quad (3)$$

The stress tensor $\boldsymbol{\sigma}(\mathbf{u})$ is symmetric, i.e. $\sigma_{12}(\mathbf{u}) = \sigma_{21}(\mathbf{u})$. The term $\dot\epsilon_d = \dot\epsilon_{kk}$ describes the divergence, and $\dot\epsilon_s = ((\dot\epsilon_{11}-\dot\epsilon_{22})^2 + 4\dot\epsilon_{12}^2)^{1/2}$ is the shear. The parameter $e = 2$ is the ratio of the major axes of the elliptic yield curve. Note that the use of the replacement pressure, $(\Delta/(\Delta+\Delta_{\min}))P$ (Hibler III and Ip, 1995) in the formulation of the VP constitutive law (2) ensures that the stress state is on an elliptic yield curve even when $\Delta \lesssim \Delta_{\min}$. The ice strength P is parameterized as $P = hP^* e^{-c^*(1-a)}$, where h is the mean thickness of the grid cell, and the constants P^* and c^* are set to $P^* = 27500$ Nm^{-2} and $c^* = 20$. For future reference we introduce the bulk and shear viscosities $\zeta = 0.5\,P/(\Delta+\Delta_{min})$ and $\eta = \zeta/e^2$.

2.1. The mEVP scheme as a pseudotime iterative scheme

The difficulty in integrating (1) is the stiff character of the stress term, which requires prohibitively small time steps in an explicit time stepping scheme. The traditional approach is either implicit (Zhang and Hibler, 1997) where viscosities are estimated at the previous nonlinear iteration and several iterations are made, or explicit, through the EVP formulation (Hunke and Dukowicz, 1997; Hunke and Lipscomb, 2008) where adding a pseudo-elastic term reduces the time step limitations. A discussion of the convergence issues can be found, for instance, in Bouillon et al. (2013); Kimmritz et al. (2015) and is not repeated here.

The suggestion by Bouillon et al. (2013) is equivalent, up to details of treating the Coriolis and the ice-ocean drag terms, to formulating the mEVP method as

$$\boldsymbol{\sigma}^{p+1} - \boldsymbol{\sigma}^p = \frac{1}{\alpha}\Big(\boldsymbol{\sigma}(\mathbf{u}^p) - \boldsymbol{\sigma}^p\Big), \quad (4)$$

$$\mathbf{u}^{p+1} - \mathbf{u}^p = \frac{1}{\beta}\left(\frac{\Delta t}{m}\nabla\cdot\boldsymbol{\sigma}^{p+1} + \frac{\Delta t}{m}\mathbf{R}^{p+1/2} + \mathbf{u}_n - \mathbf{u}^p\right). \quad (5)$$

In (5), $\mathbf{R}$ sums all the terms in the momentum equation except for the rheology and the time derivative, Δt is the external time step of the sea ice model commonly set by the ocean model, the index n labels the time levels of the model time, and the index p is that of pseudotime (subcycling step number). The Coriolis term in $\mathbf{R}^{p+1/2}$ is treated implicitly in our B-grid implementation, but is explicit on the C-grid, and the ice-ocean stress term is linearly-implicit ($C_d\rho_o|\mathbf{u}_o - \mathbf{u}^p|(\mathbf{u}_o - \mathbf{u}^{p+1})$). The term $\boldsymbol{\sigma}(\mathbf{u}^p)$ in (4) implies that the stresses are estimated by (2) based on the velocity of iteration p, and $\boldsymbol{\sigma}^p$ is the variable of the pseudotime iteration. The relaxation parameters α and β in (4) and (5) are chosen to satisfy stability constraints (see Bouillon et al. (2013); Kimmritz et al. (2015)). They replace the terms $2T/\Delta t_e$ and $(\beta^*/m)(\Delta t/\Delta t_e)$, where T is the elastic damping time scale and Δt_e the subcycling time step of standard EVP formulation; the parameter β^* was introduced in Lemieux et al. (2012). If (4) and (5) are iterated to convergence, their left hand sides can be set to zero leaving the VP solution

$$\frac{m}{\Delta t}\Big(\mathbf{u}_{n+1} - \mathbf{u}_n\Big) = \nabla\cdot\boldsymbol{\sigma}(\mathbf{u}_{n+1}) + \mathbf{R}^*, \quad (6)$$

with $\mathbf{R}^* = \lim_{p\to\infty}\mathbf{R}^{p+1/2}$ and $\mathbf{u}_{n+1} = \lim_{p\to\infty}\mathbf{u}^p$. While one may introduce a convergence criterion to determine the number of iteration steps, historically, the actual number of pseudotime iterations N is selected experimentally to ensure the accuracy needed. The new velocity $\mathbf{u}_{n+1}$ at time step $n+1$ is estimated at the last pseudotime step $p = N$. The initial values for $p = 1$ are taken from the previous time step n.

2.2. Spatial discretizations

We consider discretizations on Arakawa B- and C- grids that are commonly used in sea-ice models. The positions of variables on these grids are depicted in Fig. 1. Note, that in this section (i, j) is used as mesh indices. For simplicity we use Cartesian coordinates and uniform grids with cell widths Δx_1 and Δx_2. The complete discretization on general orthogonal curvilinear grids can be found in Bouillon et al. (2009) and Losch et al. (2010). For convenience we introduce the notation

$$\delta_1\phi_{i,j} = \phi_{i,j} - \phi_{i-1,j}, \qquad \delta_2\phi_{i,j} = \phi_{i,j} - \phi_{i,j-1},$$
$$\overline{\phi_{i,j}}^1 = (\phi_{i,j} + \phi_{i+1,j})/2, \qquad \overline{\phi_{i,j}}^2 = (\phi_{i,j} + \phi_{i,j+1})/2$$

for a quantity ϕ at a cell with index (i, j). An expression of the form $\overline{\phi_{i,j}}^{1,2}$ defines the successive application of both directional averaging operators on ϕ. Note, that the location of the discretized derivatives depends on the respective grid arrangement of variables.