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Simulation of evolution of gravity wave groups with moderate steepness

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ABSTRACT

The evolution of nonlinear deep-water wave groups in one-horizontal dimension is studied. The strongly nonlinear Green–Naghdi (GN) models are used to simulate this phenomenon. There are different levels of the GN models depending on the different velocity assumption used for the vertical structure of the flow field, such as GN-1, GN-2, and so forth. In this work, we use both the GN-3 and GN-4 models to do the simulations. Calculations are done for three different numbers of waves per group (or packet) as the number of envelope solitons depends on the number of waves per group (N). The numerical results show that the GN-3 model can give the converged GN results for the cases tested here. We conduct a series of physical experiments to investigate the evolution of wave groups. We also use other's experimental data and present laboratory measurements to compare the data with the predictions of the GN models and show that good agreement is obtained overall.

1. Introduction

The wave group evolution is an important topic in which there are many phenomena remaining undiscovered related to modulation of wave trains or packets.

In many experiments, the modulation of wave packets has been observed. Most of these works are based on the unidirectional waves. Su (1982) conducted a series of experiments on strongly nonlinear gravity-wave groups in deep water in a long towing tank. In their experiments, different nonlinearity and different N (N means the number of waves per group) are considered. As much as $N = 60$ was studied in the experiments. Tulin and Waseda (1999) conducted experiments on long-time evolution of wave groups by unstable three-wave systems. They discovered that the recurrent modulation periodically increases and decreases and this agrees with the results of the experimental study of Lake et al. (1977). Hwung et al. (2007) also investigated the long-time evolution of waves by means of a uniform wave with an imposed sideband wave. They found out that the amplitudes of the fastest growth sidebands exhibit a symmetric exponential growth until the onset of wave breaking, after which the phenomenon of frequency shifting will occur. Galchenko et al. (2010) and Tian and Choi (2013) conducted experimental studies of various effects, such as breaking effects and wind force, on the evolution of deep water wave groups.

There are many numerical studies based on nonlinear Schrödinger (NLS) equations. Shemer et al. (2002) studied the spatial evolution of nonlinear narrow-spectrum deep-water wave groups by using laboratory and computational method. The computation is based on the unidirectional Zakharov equation. It is accurate to the 3rd order in nonlinearity, as is the cubic Schrödinger equation (CSE). But contrary to the CSE, the Zakharov equation is free from any restriction on spectral width. Adcock and Taylor (2009) investigated the nonlinear evolution of unidirectional spread wave groups on deep water by using the NLS equation. They presented approximately analytical results for the evolution of one-dimensional localized wave groups, and compared the numerical results with Bateman et al. (2001)'s nonlinear theory.

The high-order spectral (HOS) method is another good model to study surface wave evolution. Dommermuth and Yue (1987) simulated the evolution of wave packets by using HOS method and compared their results with the experimental measurements of Su (1982). Mori and Yasuda (2002) investigated the effects of nonlinear interactions on wave groups by comparing the results of HOS method with the second-order approximate equations. The HOS method is better in the description of generation of high-crest waves.

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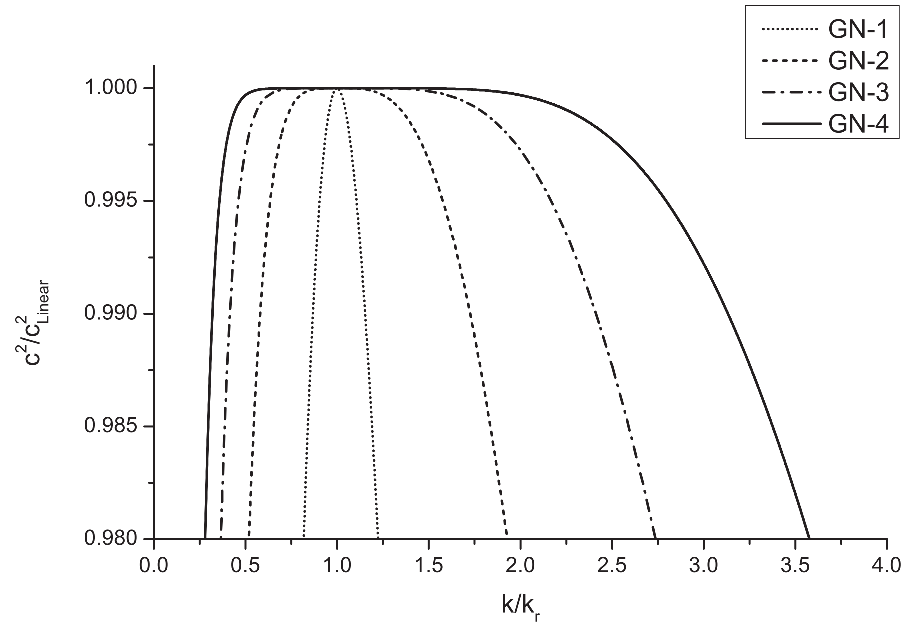

Fig. 1. Comparison of the linear dispersion relation of different levels of the GN model in deep water.

Neither the HOS method nor the NLS equations is a strongly nonlinear model. In this work, we use a strongly nonlinear numerical model called the Green–Naghdi (GN) model to investigate the evolution of uniform wave groups in deep water. The GN equations satisfy the nonlinear free-surface boundary conditions exactly. The only approximation made is on the vertical structure of the velocity field by a shape function along the water column. The governing equations of the GN models are the depth-integrated form of Euler's equations. There are GN equations for deep-water waves and shallow-water waves based on different velocity field assumption. This paper focuses on the GN equations for deep-water waves. The deep water GN models are classified into different levels, such as GN-1, GN-2, ... , and so forth, based on these approximated functions of the velocity field over the water column. Webster and Kim (1991) and Webster (2009) used the GN-3 equations to analyze large amplitude, deep-water waves in the time domain. They did not check the self-convergence of the GN models by using the GN-4 model.

The main goal of this paper is to study whether the GN model is satisfactory for modeling modulation instabilities in unidirectional wave-trains and present the self-convergence tests by comparing different levels of the GN equations. In doing so, we also have conducted experiments on the evolution of uniform wave groups of different wave steepness, wave lengths and wave heights. The present and Su (1982)'s experimental data are used to compare with the predictions of the GN equations in this work.

In Section 2, the GN equations for deep water waves are introduced. Section 3 provides the linear dispersion relation of the GN equations. Section 4 introduces the algorithm used in this work. The boundary conditions are discussed in Section 5. The numerical test cases calculated by the GN equations are presented in Section 6. These are followed by the conclusions we reached in Section 7.

2. Deep water Green–Naghdi (GN) model

In one horizontal dimension, x, the Cartesian coordinate system has its origin at the mean water level, and the vertical axis, z is taken as positive against gravity. The horizontal and the vertical velocity components are represented by $u(x, z, t)$ and $w(x, z, t)$, respectively, where t denotes time. The free surface is indicated by $z = \beta(x, t)$. We assume here that the fluid is incompressible and inviscid.

In the entire flow field, the horizontal and vertical velocities should satisfy the continuity equation for an incompressible fluid

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0. \tag{1}$$

And the conservation equations of momentum can be expressed as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{2a}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial z} + \rho g\right), \tag{2b}$$

where ρ is the density of the fluid and g the gravitational acceleration.

The kinematic free surface condition is given by

$$w - \frac{\partial \beta}{\partial t} - u\frac{\partial \beta}{\partial x} = 0, \qquad z = \beta(x, t). \tag{3}$$

The bottom boundary condition is satisfied in infinite water depth as $(u, w) \to 0$ as $z \to -\infty$. And the atmospheric pressure on the free surface, $\hat{p}(x, t)$, can be regarded as negligible without loss in generality, so that the dynamic free surface boundary condition is that the fluid pressure is $p(x, z = \beta, t) = 0$.

In the GN model, the horizontal and vertical velocities can be expressed approximately as

$$u(x, z, t) = \sum_{n=0}^{K-1} u_n(x, t)\lambda_n(z), \tag{4a}$$

$$w(x, z, t) = \sum_{n=0}^{K-1} w_n(x, t)\lambda_n(z), \tag{4b}$$

where the shape functions, $\lambda_n(z)$, could be chosen as $e^{k_r z}z^n$ (Webster and Kim, 1991) in deep water, $u_n(x, t)$ and $w_n(x, t)$ are the velocity coefficients; they are functions of time and horizontal