Ocean Modelling 90 (2015) 44–56

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

The Neutral Density Temporal Residual Mean overturning circulation

A.L. Stewart[a,*], A.F. Thompson[b]

[a] Department of Atmospheric and Oceanic Sciences, University of California, Los Angeles, CA 90095, USA
[b] Environmental Science and Engineering, California Institute of Technology, 1200 E. California Blvd., Pasadena, CA 91125, USA

ARTICLE INFO

Article history:
Received 19 September 2014
Revised 23 March 2015
Accepted 25 March 2015
Available online 02 April 2015

Keywords:
Transformed residual mean
Neutral density
Ocean overturning
Antarctic circulation

ABSTRACT

Diagnosis of the ocean's overturning circulation is essential to closing global budgets of heat, salt and biogeochemical tracers. This diagnosis is sensitive to the choice of density variable used to distinguish water masses and identify transformations between them. The oceanographic community has adopted neutral density for this purpose because its isopycnal slopes are approximately aligned with neutral slopes, along which ocean flows tend to be confined. At high latitudes there are often no tenable alternatives because potential density varies non-monotonically with depth, regardless of the reference pressure. However, in many applications the use of isoneutral fluxes is impractical due to the high computational cost of calculating neutral density. Consequently neutral density-related diagnostics are typically not available as output from ocean models.

In this article the authors derive a modified Temporal Residual Mean (TRM) approximation to the isoneutral mass fluxes, referred to as the Neutral Density Temporal Residual Mean (NDTRM). The NDTRM may be calculated using quantities that are routinely offered as diagnostic output from ocean models, making it several orders of magnitude faster than explicitly computing isoneutral mass fluxes. The NDTRM is assessed using a process model of the Antarctic continental shelf and slope. The onshore transport of warm Circumpolar Deep Water in the model ocean interior approximately doubles when diagnosed using neutral density, rather than potential density. The NDTRM closely approximates these explicitly-computed isoneutral mass fluxes. The NDTRM also exhibits a much smaller error than the traditional TRM in regions of large isoneutral temperature and salinity gradients, where nonlinearities in the equation of state diabatically modify the neutral density.

1. Introduction

The meridional overturning circulation (MOC) describes the advective transport of mass, heat, salt, and biogeochemical tracers globally between the major ocean basins (Talley, 2013). The importance of this circulation for climate and biogeochemical cycles has motivated attempts to determine a global overturning streamfunction (e.g. Lumpkin and Speer, 2007) and the construction of an array of instruments to continuously monitor the MOC in the North Atlantic (Johns et al., 2011). It is difficult in general to characterize the mean paths of fluid parcels, or "streamlines," in a turbulent flow like the ocean or atmosphere, so there is an ongoing effort to develop accurate methods for estimating the MOC. For example Döös et al. (2012) and Zika et al. (2012) have recently proposed a description of the global MOC using a streamfunction with temperature and salinity as coordinates, in which motion along streamlines correspond directly to water mass transformations.

One case in which it is possible to exactly characterize volume transports is when the fluid is constrained to flow within a stack of non-intersecting material surfaces, a criterion that is approximately satisfied by isopycnals in the ocean. It has long been known that calculating the "Eulerian-mean" MOC, i.e. using the time- and zonal-averaged velocity, yields misleading results (Döös and Webb, 1994; Nurser and Lee, 2004; Hirst et al., 1996). Calculating volume fluxes within isopycnal surfaces yields a much more accurate estimate of the transport, accounting for the "eddy" component of the MOC (Marshall and Radko, 2003; Zika et al., 2013). In principle it is possible to compute the MOC from the isopycnal volume fluxes in any predictive ocean model or reanalysis product. To avoid aliasing this calculation requires output of the model velocities and state variables with a time interval much shorter the typical turnover time of mesoscale eddies (Ballarotta et al., 2013). However, the frequency may be limited by digital storage constraints and the burden of post-processing such a large volume of data. These constraints may be circumvented,

* Corresponding author.
E-mail address: astewart@atmos.ucla.edu (A.L. Stewart).

http://dx.doi.org/10.1016/j.ocemod.2015.03.005
1463-5003/

and the accuracy of the calculation improved, by computing and averaging the isopycnal fluxes during numerical integration of the model. These fluxes are typically available as output in models that use an isopycnal vertical coordinate (e.g. Hallberg and Rhines, 1996), and are also available in some non-isopycnal models like the MIT general circulation model MITgcm, (Marshall et al., 1997b,a).

When neither high-frequency model output nor isopycnal fluxes are available, an alternative approach to computing the MOC is to approximate the isopycnal fluxes using the Transformed Eulerian Mean TEM, (Plumb and Ferrari, 2005; Nurser and Lee, 2004) or Temporal Residual Mean TRM, (McDougall and McIntosh, 1996, 2001). At leading order in the isopycnal fluctuations, the TEM/TRM streamfunctions may be computed solely from the averages of the velocity, the density, and the product of the velocity and the density (Wolfe, 2014). They therefore incur a much smaller computational cost than directly calculating the fluxes between isopycnal surfaces.

The volume fluxes calculated between isopycnal surfaces will depend upon the choice of density variable. Parcels of ocean water are strongly constrained to follow so-called neutral surfaces, as parcels moving along these surfaces feel no buoyant restoring forces (McDougall, 1987). It is therefore desirable that the isopycnal slopes of the chosen density variable should coincide with neutral slopes (Eden and Willebrand, 1999; McDougall and Jackett, 2005). Mathematically, neutral surfaces are locally perpendicular to the vector $\boldsymbol{A}$, where

$$\boldsymbol{A} = \beta \nabla S - \alpha \nabla \theta. \tag{1}$$

Here S is the (practical) salinity, θ is the potential temperature, β is the saline contraction coefficient, and α is the thermal expansion coefficient. Neutral surfaces are in fact only well-defined if $\boldsymbol{A}$ has zero helicity, i.e. $\boldsymbol{A} \cdot \nabla \times \boldsymbol{A} = 0$ (McDougall, 1987). The helicity of $\boldsymbol{A}$ is generally non-zero due to the nonlinearity of the equation of state for seawater. However, the helicity does tend to be small because the ocean spans a relatively small volume in temperature/salinity/pressure space (McDougall and Jackett, 2007), so for practical purposes an approximate set of neutral surfaces can be constructed.

There are several possible candidates for the density variable that should be used to define isopycnal fluxes. Potential density carries the benefit of being materially conserved away from regions of direct heating or freshwater forcing, except due to diffusion of temperature and salinity. However, the slopes of potential density surfaces can differ substantially from neutral slopes (McDougall, 1987). This is most pronounced at high latitudes, where potential density may no longer vary monotonically with depth; in such regions, using potential density to compute isopycnal fluxes would result in a substantial loss of information. de Szoeke et al. (2000) constructed an "orthobaric density", by empirically removing the dependence of *in situ* density. However, the corresponding isopycnal slopes still differ substantially from neutral slopes, and orthobaric density also varies non-monotonically with depth in some regions (McDougall and Jackett, 2005). Jackett and McDougall (1997) proposed a "neutral density" variable γ that is constructed to be constant along approximate neutral surfaces. This construction should ensure monotonicity with depth, though neutral density is only quasi-materially conserved; the nonlinearity of the equation of state precludes the existence of any density variable that is both isoneutral and materially conserved (McDougall and Jackett, 2005).

Using neutral density to compute the MOC provides an accurate representation of the overturning circulation at all ocean depths, in contrast to potential density (Hirst et al., 1996). However this also magnifies the difficulties associated with calculating isopycnal fluxes, described above, due to the computational expense incurred in calculating the neutral density at many time intervals. Consequently there are currently no ocean models that maintain neutral density as a state variable, offer neutral density as an output product, nor compute mass/volume fluxes between neutral density surfaces. Thus it is not even possible to compute a TEM or TRM streamfunction based on neutral density, as this requires the average of the product of velocity and density $\overline{\boldsymbol{u}\gamma}$. In principle the neutral density and isopycnal volume fluxes could be computed during model integration, or during post-processing from high-frequency model output. However, we will show in Sections 3 and 4 that the computational cost of calculating neutral density makes such approaches inefficient, especially if applied to global models running at eddy-permitting resolution or higher.

In this paper we propose an efficient method of approximating volume fluxes between neutral density surfaces using a modified TRM that uses only quantities typically available as output from z-coordinate and terrain-following ocean models (e.g. Marshall et al., 1997b,a; Haidvogel et al., 2008). More precisely, we assume that the time-mean salinity $\overline{S}$, potential temperature $\overline{\theta}$ and pressure $\overline{p}$ are available, along with the time-mean of the product of the velocity and state variables, e.g. $\overline{\boldsymbol{u}\theta}$. This approximate TRM streamfunction, which we refer to as the Neutral Density Temporal Residual Mean (NDTRM) is derived in Section 2. Then in Section 3 we test the accuracy of the NDTRM using a process model of the Antarctic continental shelf and slope. Finally, in Section 4 we discuss our findings and provide concluding remarks.

2. Derivation of the Neutral Density Temporal Residual Mean (NDTRM) streamfunction

2.1. Volume fluxes within neutral density layers

In the interest of a self-contained derivation, we begin with a brief review of McIntosh and McDougall (1996) and McDougall and McIntosh (2001), who derive expressions for the TRM using neutral density as a state variable. We restrict our attention to a Cartesian geometry to simplify our presentation, defining the overbar $\overline{\bullet}$ as a time average over many eddy rotation timescales in a statistically steady state.

Consider a neutral surface $\gamma = \gamma_0$ at a lateral position (x_0, y_0). The depth of the neutral surface is $z = z_0(\gamma_0, t)$. The overturning streamfunction is defined as the lateral volume flux between $z = z_0$ and the ocean surface (e.g. Döös and Webb (1994)), which for simplicity we regard as a rigid lid at $z = 0$,

$$\boldsymbol{\Psi}(x_0, y_0, \gamma_0) = \overline{\int_{z=z_0(\gamma_0)}^{z=0} \boldsymbol{u}\, \mathrm{d}z}. \tag{2}$$

Here $\boldsymbol{u} = (u, v)$ is the horizontal velocity vector. Note that $\boldsymbol{\Psi}$ is a function of lateral position and neutral density: vertical gradients in this streamfunction correspond to lateral velocities within density classes, whilst lateral gradients correspond to diapycnal fluxes.

We proceed under the assumption that deviations of the isopycnal depth z_0 from the mean are small compared to the ocean depth, measured by $\varepsilon \sim z_0'/H \ll 1$. We similarly assume that the perturbations of the velocity, potential temperature, salinity and neutral density are small relative to vertical changes in their respective means, e.g. $\gamma'/\overline{\gamma}_z H = \mathcal{O}(\varepsilon)$. This holds if such deviations are associated with the eddy-induced vertical heaving of the isopycnals. We may then expand (2) via a Taylor expansion,[1]

[1] Formally such asymptotic approximations should be made in dimensionless variables. Throughout the manuscript we retain dimensional variables for clarity and for consistency with previous derivations of the TRM (McIntosh and McDougall, 1996, 2001; Nurser and Lee, 2004,; Wolfe, 2014).