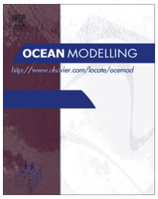
Ocean Modelling 87 (2015) 1–19

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod



Dynamically consistent parameterization of mesoscale eddies. Part I: Simple model

Pavel Berloff *

Department of Mathematics, Imperial College London, London, United Kingdom

ARTICLE INFO

Article history:
Received 24 July 2014
Received in revised form 24 November 2014
Accepted 2 December 2014
Available online 17 January 2015

Keywords:
Geophysical fluid dynamics
Parameterization of mesoscale eddies

ABSTRACT

This work aims at developing a framework for dynamically consistent parameterization of mesoscale eddy effects for use in non-eddy-resolving ocean circulation models. The proposed eddy parameterization framework is successfully tested on the classical, wind-driven double-gyre model, which is solved both with explicitly resolved vigorous eddy field and in the non-eddy-resolving configuration with the eddy parameterization replacing the eddy effects. The parameterization locally approximates transient eddy flux divergence by spatially localized and temporally periodic forcing, referred to as the *plunger*, and focuses on the linear-dynamics flow solution induced by it. The nonlinear self-interaction of this solution, referred to as the *footprint*, characterizes and quantifies the induced cumulative eddy forcing exerted on the large-scale flow. We find that spatial pattern and amplitude of the footprint strongly depend on the underlying large-scale and the corresponding relationships provide the basis for the eddy parameterization and its closure on the large-scale flow properties. Dependencies of the footprints on other important parameters of the problem are also systematically analyzed. The parameterization utilizes the local large-scale flow information, constructs and scales the corresponding footprints, and then sums them up over the gyres to produce the resulting eddy forcing field, which is interactively added to the model as an extra forcing. The parameterization framework is implemented in the simplest way, but it provides a systematic strategy for improving the implementation algorithm.

1. Introduction

Mesoscale oceanic eddies populate nearly all parts of the global ocean and play important roles in maintaining the oceanic general circulation (e.g., McWilliams (2008)). The most straightforward, but also the most computationally intensive and, thus, unfeasible, way of accounting for the eddy effects on the large-scale circulation is resolving them dynamically with eddy-resolving ocean general circulation models (GCMs). This brute-force approach requires the computational grid resolution of about 1 km, which makes it feasible only for relatively short-time simulations, whereas the Earth system and climate change modeling routinely require much longer simulations over centuries and millenia. The only way to afford these time scales, while simulating the ocean in qualitatively correct way, is to *parameterize* the important eddy effects with simple and affordable but still accurate models embedded in the GCMs. In this context an eddy parameterization is a parametric mathematical model to be used in some coarse-grained, reduced-dynamics ocean circulation model. Ideally, the parameters involved are to be related to the explicitly resolved, large-scale circulation properties, thus, resulting in a turbulence closure for the eddies. Over the last few decades, the search for suitable eddy parameterizations remains a challenging theoretical topic with clear practical dimension.

In this paper, we propose, investigate and test a novel eddy parameterization framework that can stimulate both theoretical and practical advances. The main essence of the new parameterization is its focus on the *transient fluctuations* of the geostrophic eddy fluxes affecting the large-scale flow. The other essence is the dynamical consistency of the proposed framework. We aim more at the conceptual and dynamical foundation for the parameterization, rather than at the development of its final and polished algorithm. Overall, our results are fundamental, encouraging in terms of successful tests of the simplest initial implementation, and providing the framework for further systematic research and improvements.

Plan of the presentation is the following. In Section 1.1 we discuss the underlying philosophy and the background literature, as well as the implemented research strategy. The dynamical ocean model in which the parameterization is implemented and tested

---

* Address: Imperial College London, Department of Mathematics, Huxley Bldg., London SW7 2AZ, United Kingdom.

E-mail address: p.berloff@imperial.ac.uk

http://dx.doi.org/10.1016/j.ocemod.2014.12.008
1463-5003/

is described in Section 1.2. The nonlinear eddy dynamics explicitly simulated by the eddy-resolving ocean model configuration is analyzed in Section 2. In Sections 3 and 4 we study linear-dynamics flow responses to localized transient forcings, which imitate the actual eddy flux divergences acting on the large-scale flow, and find their dependencies on the large-scale flow. These analyses provide the theoretical underpinnings and principles of the eddy parameterization, which is eventually implemented and tested in Section 5. In Section 6 we summarize the parameterization framework, discuss the results and outline further research avenues.

1.1. Background and statement of the problem

The most common approach for parameterizing mesoscale eddy effects is (turbulent) *eddy diffusion*, which assumes that the eddies transport the corresponding flow property down its large-scale gradient. The eddy viscosity[1] is implemented in any GCM, and the eddy buoyancy diffusion, which parameterizes ubiquitous baroclinic instability processes (Gent and McWilliams, 1990), is used in most of the GCMs. The latter parameterization leads to substantial model improvements in many parts of the global ocean, especially in the Southern Ocean. Most of the diffusive parameterization theories focus on estimating various eddy diffusivity coefficients and relating them to the large-scale flow properties; for example, by invoking local linear-stability analysis (e.g., Eden (2011)) or by enforcing consistency with the physical conservation laws (e.g., Marshall et al. (2012), Ivchenko et al. (2013)). Very few studies attempt to challenge the very nature of the diffusive approach.

The main drawbacks of the diffusive approach are the following. First, the down-gradient assumption is often valid, especially for passive tracers, but it completely breaks down in the "negative eddy viscosity" and "negative eddy diffusivity" situations (Starr, 1968) occurring with active tracers, such as momentum, buoyancy and potential vorticity (PV). For example, in the eastward jet extension of the western boundary currents, the eddies flux PV up the large-scale PV gradient (Berloff et al., 2005b). Second, it is often assumed that the eddy diffusivity (and viscosity) coefficient is isopycnally isotropic and spatially homogeneous, although there is massive evidence against this assumption (e.g., Rypina et al. (2012)). Third, it is not usually understood how to relate an eddy diffusivity to the large-scale flow, hence, the diffusive parameterization remains unclosed and, thus, incomplete.

Despite intrinsic limitations of the eddy diffusion parameterizations, it is popular not only due to its mathematical simplicity, but also due to the lack of alternative theoretical ideas. An emerging theoretical alternative to the diffusion is to rely on *random* rather than deterministic representation of the diverging eddy fluxes. The main potential advantage of this approach is its capability to account for the negative-diffusivity eddy effects, which can not be modeled as diffusion due to the mathematical ill-posedness. In this case the randomness is justified by the observed, highly transient and structurally complicated pattern of the eddy fluxes. Although detailed observations of the oceanic eddy fluxes are problematic and scarce, the eddy-resolving GCMs robustly simulate the eddy flux divergence characterized by complex spatio-temporal patterns and by large transient fluctuations around small time-mean values (e.g., Li and von Storch (2013)). Can fluctuations of the eddy flux divergence be modeled as a random-forcing process, and can this approach eventually parameterize the important eddy effects? The foregoing, classical homogeneous-turbulence approach suggests to replace small-scale spectral nonlinear interactions by a statistically similar random forcing (e.g., Herring (1996)). The oceanic mesoscale turbulence is spatially inhomogeneous, therefore, the spectral approach should be reformulated in the physical space. However, this will not mitigate the main problems of the whole approach: (1) constraining random forcing with some physical principles and (2) relating parameters of the random forcing to the large-scale flow fields. Presumably, random forcing could be calibrated from the eddy-resolving simulations (e.g., Berloff et al. (2005a,b)) or estimated from the observations (assuming that we know what and where to observe). The other problem is (3) finding a proper large-scale circulation model compatible with the random forcing. Suggestions for such a model range from linear (e.g., see review by Penland (2007)) to nonlinear (Berloff et al., 2005a; Porta Mana and Zanna, 2014; Jansen and Held, 2014), depending on the objectives. In general the underlying model has to be fluid-dynamical and non-eddy-resolving, but severely truncated dynamics can be also used for specific purposes, such as modeling the large-scale low-frequency variability (e.g., Kravtsov et al. (2005)).

There are several precursors to the present work that involve both oceanic gyres and random-forcing approach. The first precursor is a sequence of papers (Berloff et al., 2005a,b; Berloff et al., 2007), in which the eddy-resolving solutions are used for constructing and constraining a family of random-forcing parameterizations incorporated in the non-eddy-resolving models and successfully tested. Nevertheless, the proposed framework has two shortcomings. First, relations between the random forcing and the large-scale flow properties remain poorly understood, thus, hampering the complete closure. Second, the randomly forced flow dynamics remains poorly understood, thus, hampering the physical understanding. The second precursor to our work is recent study by Porta Mana and Zanna (2014), in which the random forcing is shaped by the probability density function calibrated on the eddy-resolving simulations and conditioned on the explicitly resolved large-scale flow properties. Our present work compliments and extends the above-described studies. It is novel in the sense that, not only it illuminates the dynamical connections between transient eddies and their large-scale effects, but also it develops and implements the corresponding eddy parameterization.

The central building block of our approach is analysis of the linear-dynamics responses to spatially localized and temporally periodic forcing function referred to as *plunger* as representing an elementary transient action by the eddies. Flow response to a plunger can be treated as the convolution of the Green's functions of the problem. The temporal periodicity of the plunger is an interim simplification that can be later upgraded to more general, random but time-correlated process (e.g., Berloff and McWilliams (2003)). The proposed approach is radically different from the classical spectral random forcing, because spatially localized forcing is spectrally broad-band, with phase-correlated harmonics.

The simplest conceptual model of the nonlinear rectification of plunger-induced flows is sometimes referred to as the "beta-plane plunger", and its main aspect is emergence of (1) a rectified eastward jet at the directly forced latitudes and (2) westward return currents outside of them (Whitehead, 1975; Haidvogel and Rhines, 1983; Waterman and Jayne, 2011; Waterman and Jayne, 2012). The eastward/westward flows are driven by the diverging upgradient/downgradient eddy PV fluxes, and in this process both the nonlinearity and the background PV gradient are fundamentally important. In the simplest set-up, the background PV gradient is uniform and set by the beta-plane approximation. In this paper we systematically explore the plunger dynamics before incorporating it into the parameterization.

Our approach is the following. We resorted to the quasigeostrophic (QG) dynamics of the classical double-gyre model, which is solved both in the eddy-resolving and non-eddy-resolving con-

---

[1] Here, the term "viscosity" applies to the momentum, and the term "diffusivity" applies to all other scalar properties.