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Approximate deconvolution large eddy simulation of a stratified two-layer quasigeostrophic ocean model

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ABSTRACT

We present an approximate deconvolution (AD) large eddy simulation (LES) model for the two-layer quasigeostrophic equations. We applied the AD-LES model to mid-latitude two-layer square oceanic basins, which are standard prototypes of more realistic stratified ocean dynamics models. Two spatial filters were investigated in the AD-LES model: a tridiagonal filter and an elliptic differential filter. A sensitivity analysis of the AD-LES results with respect to changes in modeling parameters was performed. The results demonstrate that the AD-LES model used in conjunction with the tridiagonal or differential filters provides additional dissipation to the system, allowing the use of a smaller eddy viscosity coefficient. Changing the spatial filter makes a significant difference in characterizing the effective dissipation in the model. It was found that the tridiagonal filter introduces the least amount of numerical dissipation into the AD-LES model. The differential filter, however, added a significant amount of numerical dissipation to the AD-LES model for large values of the filter width. All AD-LES models reproduced the DNS results at a fraction of the cost within a reasonable level of accuracy.

1. Introduction

The investigation of characteristics of forced-dissipative general circulation models is of primary importance in developing our understanding of the large-scale nonlinear motions of geophysical flows. As one of the main circulation sources, winds drive the general circulation associated with the subtropical and subpolar gyres, which can be identified with the strong, persistent, sub-tropical and sub-polar western boundary currents in the North Atlantic Ocean (the Gulf Stream and the Labrador Current) and North Pacific Ocean (the Kuroshio and the Oyashio Currents) and sub-tropical counterparts in the southern hemisphere (Stommel, 1972; McWilliams, 2006). One of the major similarities between the various ocean basins is the asymmetry of the gyres: strong western boundary currents and weaker flow in the interior; weak and shallow eastern boundary currents. The most obvious motivation for being interested in forced-dissipative wind-driven ocean circulation is the connection between ocean currents and climate dynamics (Ghil et al., 2008).

The wind-driven circulation in an enclosed, midlatitude rectangular or square basin is a classical problem, studied extensively bymodelers (Allen, 1980; Holland and Rhines, 1980; Griffa and Salmon, 1989; Vallis, 2006; Miller, 2007). Various models are derived from the full-fledged equations of geophysical flows, cBoussinesq equations (BEs) or the primitive equations (PEs), to guide the theoretical studies on boundary currents, alternating zonal flows, or jet formations, as well as to identify some key issues related to the relative insensitivity of the model dynamics to the changes of parameters that is closely linked to a dynamical system point of view (Speich et al., 1995; Meacham, 2000; Chang et al., 2001; Nauw et al., 2004; Dijkstra, 2005; Dijkstra and Ghil, 2005). The quasigeostropic (QG) model is a simplification of the primitive equation model that retains many of the essential features of geophysical fluid flows. Details of the mathematical and physical approximations may be found in standard textbooks on geophysical fluid dynamics, such as Pedlosky (1987), Vallis (2006) and McWilliams (2006). The main assumptions that go into the QG models are: the hydrostatic balance, the β-plane approximation, the geostrophic balance, and the eddy viscosity parameterization.

The one-layer QG model, sometimes called the barotropic vorticity equation (BVE), represents one of the most commonly used mathematical models for these types of geostrophic flows with various dissipative and forcing terms (Majda and Wang, 2006; Vallis, 2006; Nadiga and Margolin, 2001). In reality, the ocean is a stratified fluid on a rotating Earth driven from its upper surface by patterns of momentum and buoyancy fluxes (Marshall et al., 1997). While the barotropic model is not stratified, it exhibits many of the features that are observed in the stratified case. To explore some of the effects of the stratification, the one-layer barotropic equation can be extended to the 1.5-layer model, also called the reduced gravity QG model (Özgökmen et al., 2001).

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Table 1
The eddy viscosity coefficients used in QG models.

Study	Range of ν ($m^2 s^{-1}$)	Resolution
Bryan (1963)	500–10000	40 × 80
Gates (1968)	6000–10000	74 × 50
Holland and Lin (1975)	330	50 × 50
Jiang et al. (1995)	300	50 × 100
Özgökmen and Chassignet (1998)	50	151 × 151
Berloff and McWilliams (1999)	400–1600	256 × 256
Sura et al. (2001)	200	120 × 120
Berloff et al. (2009)	100	512 × 256
Tanaka and Akitomo (2010)	100	500 × 500

There are two layers in this model, but the second layer is infinitely deep and at rest (passive), and the dynamics are effectively barotropic. The two-layer model takes the next step in increasing the complexity of stratification by adding a second dynamically active layer (Holland, 1978; Özgökmen and Chassignet, 1998; Berloff and McWilliams, 1999; DiBattista and Majda, 2001; Berloff et al., 2009). The dynamics in this model include the first baroclinic modes. The complexity of the models could be increased by adding more active layers, resulting in the N-layer models (Siegel et al., 2001), which, in turn, yield the three dimensional primitive equations when N goes to infinity (McWilliams, 2006). In this study, we use the *two-layer QG (QG2)* model.

Geophysical turbulence is strongly affected by the planetary vorticity, the variation of the Coriolis parameter with latitude, the so-called β effect (Maltrud and Vallis, 1991; Smith et al., 2002; Chen et al., 2003). The inverse cascade typically occurring in pure two-dimensional turbulence, in this case preferentially transfers small-scale energy towards zonal modes; the resulting flow is then anisotropic and characterized by a strong interaction between waves and turbulence, and is known as the arrest of the inverse energy cascade (Rhines, 1975; Sukoriansky et al., 2007; Espa et al., 2008; San and Staples, 2013b). Rhines (1975) explained the emergence of flow anisotropy and the organization of a banded pattern of alternating zonal currents, or jets, due to Rossby wave dynamics in terms of a competition between nonlinear and β terms in the barotropic vorticity equation. Under the effects of planetary rotation, Rossby waves dominate turbulent motions prohibiting the triad interactions, and arrest the inverse energy cascade when the scale of motions becomes larger than a critical value, later known as the Rhines scale (Tanaka and Akitomo, 2010).

Along with the Rhines scale which is a measure of the strength of nonlinear interactions, another important scale for determining the dynamics of the large scale motions in the ocean is the Munk scale (Munk, 1950), which corresponds to the dissipative behavior of the system and can be linked to the Reynolds number. Although the water molecular viscosity is around 10^{-6} m^2 s^{-1}, the one- and two-layer QG models use viscosities on the order of 10^2 m^2 s^{-1}. This is called *eddy viscosity (EV)* parameterization, and is used because the horizontal scale of the ocean basin is much larger than the effective scale for molecular diffusion. An impractically fine resolution would be necessary if the ocean models were to resolve the full spectra of turbulence down to the Kolmogorov scale. Thus, the viscosity coefficients employed in the QG models typically remain much greater than the molecular viscosity (Campin et al., 2011). The eddy viscosities generally used in the oceanic models are summarized in Table 1. The eddy viscosity parameterization used in the QG models plays a crucial role in the dynamics of the problem. Indeed, Berloff and McWilliams (1999) studied the wind-driven circulation in a three-layer QG model for varying values of the eddy viscosity coefficient in a square oceanic basin. For $\nu = 1200$ m^2 s^{-1} an asymmetric steady state was found. When the eddy viscosity coefficient was decreased, the flow first displayed a variability characterized by the presence of interior Rossby waves. At $\nu = 1000$ m^2 s^{-1}, the flow regime showed a quasi-periodic variability. At a smaller eddy viscosity coefficient, starting from $\nu = 800$ m^2 s^{-1}, the flow regime was chaotic and showed a persistent eastward jet penetration by fluctuating between two preferred states, one of which corresponds to a low energy state and a long eastward jet, and the other to a high energy state and a short jet. The study of Berloff and McWilliams (1999) clearly shows that different EV coefficients can result in different dynamics of the QG models. Thus, a natural question is “What EV coefficient should be used in the QG models?” The EV coefficients summarized in Table 1 seem to convey, at first glance, a confusing message: they vary by as much as an order of magnitude. At a closer look, however, Table 1 clarifies this issue: With the ever increasing computational power, the mesh size used in numerical simulations with the QG models constantly decreases and allows the use of smaller EV coefficients. The development of a rigorous, mathematical understanding and subsequent modeling strategy for the eddy viscosity coefficients (see Table 1) is the “elephant in the room,” one of the major unsolved problems in ocean modeling (Visbeck et al., 1997; Campin et al., 2011; Majda and Wang, 2006; Cushman-Roisin and Beckers, 2009; Vallis, 2006). Although addressing this grand challenge is beyond the scope of this report, we do address the intimate relationship between the EV coefficients and the numerical resolution employed by the QG models.

To capture the under-resolved flow, i.e., the flow in the regions where the grid size becomes greater than the specified Munk scale, *large eddy simulation (LES)* appears as a natural choice. Most of the LES models have been developed for three-dimensional turbulent flows, such as those encountered in engineering applications (Sagaut, 2006; Berselli et al., 2006). These LES models fundamentally rely on the concept of the forward energy cascade and so their extension to geophysical flows is beset with difficulties. The effective viscosity values in oceanic models are much greater than the molecular viscosity of seawater, hence a uniform eddy viscosity coefficient is generally used to parameterize the unresolved, subfilter-scale effects in most oceanic models (McWilliams, 2006; Vallis, 2006). LES models specifically developed for two-dimensional turbulent flows, such as those in the ocean and atmosphere, are relatively scarce (Fox-Kemper and Menemenlis, 2008; Awad et al., 2009; Özgökmen et al., 2009; Chen et al., 2011), at least when compared to the plethora of LES models developed for three-dimensional turbulent flows. Holm and Nadiga (2003) combined the uniform eddy viscosity parameterization with the alpha regularization LES approach to capture the under-resolved flow where the grid length becomes greater than the specified Munk scale of the problem. In that work, the structural alpha parameterization was tested on the barotropic vorticity equation (BVE) in an ocean basin with double-gyre wind forcing, which displays a four-gyre mean ocean circulation pattern. It was found that the alpha models provide a promising approach to LES closure modeling of the barotropic ocean circulation by predicting the correct four-gyre circulation structure for under-resolved flows.

San et al. (2011) put forth a new LES closure modeling strategy for two-dimensional turbulent geophysical flows. The new closure modeling approach utilizes *approximate deconvolution (AD)*, which is particularly appealing for geophysical flows because of no additional phenomenological approximations to the BVE. Similar to the method suggested by Holm and Nadiga (2003), this framework also uses a Laplacian operator with a constant eddy viscosity coefficient to account for the dissipation mechanism. For a given system with eddy viscosity dissipation, the subfilter-scale contribution, however, is modeled by a non eddy viscosity AD closure approach. The AD approach can achieve high accuracy by employing repeated filtering, which is computationally efficient and easy to implement. The AD method has been used successfully in LES of three-dimensional turbulent engineering flows (Stolz and