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Modelling the effect of bottom sediment transport on beach profiles and wave set-up

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ABSTRACT

In this paper we augment the wave-averaged mean field equations commonly used to describe wave set-up and wave-induced mean currents in the near-shore zone, with an empirical sediment flux law depending only on the wave-induced mean current and mean total depth. This model allows the bottom to evolve slowly in time, and is used to examine how sediment transport affects the beach profile and wave set-up in the surf zone. We show that the mean bottom depth in the surf zone evolves according to a simple wave equation, whose solution predicts that the mean bottom depth decreases and the beach is replenished. Further, we show that if the sediment flux law also allows for a diffusive dependence on the beach slope then the simple wave equation is replaced by a nonlinear diffusion equation which allows a steady-state solution, the equilibrium beach profile.

1. Introduction

The action of shoaling waves, and wave breaking in the surf zone, in generating a wave-generated mean sea-level is well-known and has been extensively studied, see for instance the monographs by Mei (1983) and Svendsen (2006). The simplest model is obtained by averaging the oscillatory wave field over the wave phase to obtain a set of equations describing the evolution of the mean fields in the shoaling zone based on small-amplitude wave theory and then combining these with averaged mass and momentum equations in the surf zone, where an empirical formula is used for the breaking waves. These lead to a prediction of a steady set-down in the shoaling zone, and a set-up in the surf zone. This agrees quite well with experiments and observations, see Bowen et al. (1968) for instance. However, these models assume that the sea bottom is rigid, and ignore the possible effects of sand transport by the wave velocity field, and the wave-generated mean currents. Our purpose in this paper is to add an empirical model of sediment transport to the wave-averaged mean field equations and hence determine the effect of this extra term on the beach profile and the wave set-up.

There is a vast literature on sediment transport due to waves, see the recent works by Caballeria et al. (2002); Calvete et al. (2001, 2002); Garnier et al. (2006, 2008); Hancock et al. (2008); Lane and Restrepo (2007); McCall et al. (2010); Plant et al. (2001); Restrepo (2001); Restropo and Bona (1995); Roelvink et al. (2009); Ruessink et al. (2012) and Walgreen et al. (2002) and the references therein, to name just a few representative works. There are several methods to model the movement of bottom sediment by the combined action of waves and currents, and these can often be quite complicated, depending *inter alia* on the nature of the sediment, and whether the sediment is confined to the bottom boundary layer, or is suspended throughout a larger portion of the water column. Various models have been used to describe the formation of sand bars, ripples and sand waves, where it has usually been assumed that the wave field is quasi-periodic and non-breaking, see for instance the afore-mentioned articles and the review article by Blondeaux (2001). For the most part, application of these models to the near shore zone, where there is wave breaking, has been confined to numerical simulations. In particular, the effect of sediment transport on the beach profile and the wave set-up, especially in the surf zone, does not appear to have been examined in analytical detail, which is contrast to the case when there is no sediment transport where a well-established analytical theory exists, see Mei (1983) or Svendsen (2006) for instance. To remedy this, we modify the well-known wave-averaged mean field equations by a bottom boundary condition that allows for the evolution of the bottom as sediment is moved. This leads to a single extra equation in the wave-averaged mean field model to represent the time evolution of the bottom, based on a relatively simple empirical law for the bottom sediment flux, based on the sediment transport models used in similar problems in the cited references above. Although our model is two-dimensional in principle, in this study we restrict ourselves to a one-dimensional implementation, with a main focus on how sediment transport affects wave set-up and the bottom beach profile.

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In Section 2 we present the usual wave-averaged mean field equations that are commonly used in the literature, supplemented here by a bottom sediment transport term. We then examine the consequences for the beach profile and the wave set-up in Section 3. We conclude with a discussion in Section 4.

2. Formulation

2.1. Wave field

In this section we recall the wave-averaged mean flow and wave action equations that are commonly used to describe the near-shore circulation, see Mei (1983) or Svendsen (2006) for instance. We assume that the depth and the mean flow are slowly-varying compared to the waves. Then we define a wave-phase averaging operator $\langle f\rangle = \bar{f}$, so that all quantities can be expressed as a mean field and a wave perturbation, denoted by a ~ overbar. For instance,

$$\zeta = \bar{\zeta} + \tilde{\zeta}. \quad (1)$$

where ζ is the free surface elevation above the bottom located at $z = -h(\mathbf{x}, t)$. Then outside the surf zone, the representation for slowly-varying, small-amplitude waves is, in standard notation,

$$\tilde{\zeta}(\mathbf{x}, t) = a\cos\theta + O(a^2). \quad (2)$$

Here $a = a(\mathbf{x}, t)$ is the wave amplitude in the linearized theory, and we shall consistently neglect the higher-order term $O(a^2)$. $\theta = \theta(\mathbf{x}, t)$ is the phase, such that the wavenumber $\mathbf{k}$, frequency Ω are given by

$$\mathbf{k} = (k, l) = \nabla\theta, \quad \Omega = -\theta_t, \quad (3)$$

where $\nabla = (\partial_x, \partial_y)$. The local linear dispersion relation is

$$\Omega = \omega + \mathbf{k}\cdot\mathbf{U}, \quad \omega^2 = g\kappa\tanh\kappa H \quad (4)$$

where $\kappa^2 = k^2 + l^2$.

Here $\mathbf{U}(\mathbf{x}, t)$ is the slowly-varying depth-averaged mean current (see below), and $H(\mathbf{x}, t) = \bar{h}(\mathbf{x}, t) + \bar{\zeta}(\mathbf{x}, t)$ is the total fluid depth, also a slowly varying function of $\mathbf{x}, t$. Note that $h(\mathbf{x}, t)$ is a dynamic quantity, whose evolution is described in Section 2.3 below.

The basic equations governing the wave field are then the kinematic equation for conservation of waves

$$\mathbf{k}_t + \nabla\omega = 0, \quad (5)$$

which is obtained from (3) by cross-differentiation, the local dispersion relation (4), and the wave action equation for the wave amplitude

$$A_t + \nabla\cdot(\mathbf{c}_g A) = 0. \quad (6)$$

In this linearized theory, $A = E/\omega$, where $E = ga^2/2$ is the wave energy per unit mass, and $\mathbf{c}_g = \nabla_{\mathbf{k}}\cdot\omega = \mathbf{U} + c_g\mathbf{k}/\kappa, (c_g = d\omega/d\kappa)$ is the group velocity.

2.2. Mean fields

The equations governing the mean fields are obtained by averaging the depth-integrated Euler equations over the wave phase. Thus the averaged equation for conservation of mass is

$$H_t + \nabla\cdot(H\mathbf{U}) = 0. \quad (7)$$

For the velocity field we proceed in a slightly different way, that is we set

$$\mathbf{u} = \mathbf{U} + \mathbf{u}', \quad (8)$$

where $\mathbf{U}$ is defined so that the mean momentum density is given by

$$\mathbf{M} = H\mathbf{U} = \left\langle \int_{-h}^{\zeta} \mathbf{u}\, dz \right\rangle, \quad (9)$$

But now we need to note that $\mathbf{u}'$ does not necessarily have zero mean, and that $\mathbf{U}$ and $\bar{\mathbf{u}}$ are not necessarily the same. Indeed, from (8) and (9) we get that

$$\bar{\mathbf{u}} = \mathbf{U} + \langle\mathbf{u}'\rangle, \quad \text{and} \quad \left\langle \int_{-h}^{\zeta} \mathbf{u}'\, dz \right\rangle = 0.$$

However, $\mathbf{u}' = \tilde{\mathbf{u}} + O(a^2)$, so that $\langle\mathbf{u}'\rangle$ is $O(a^2)$ and it follows that, correct to second order in wave amplitude,

$$\mathbf{M} = H\bar{\mathbf{u}} + \mathbf{M}_w, \quad \text{where} \quad \mathbf{M}_w = -H\langle\mathbf{u}'\rangle = \langle\zeta\tilde{\mathbf{u}}(x, 0, t)\rangle = \frac{E}{\omega}\mathbf{k}. \quad (10)$$

The term $\mathbf{M}_w$ in (10) is called the wave momentum, and can be expressed as $\mathbf{M}_w = H\mathbf{U}_s$ where $\mathbf{U}_s$ is the Stokes drift velocity. It follows that $\mathbf{U}$ is a Lagrangian mean flow.

Averaging the depth-integrated horizontal momentum equation yields, see (Mei, 1983),

$$(H\mathbf{U})_t + \nabla\cdot(H\mathbf{U}\mathbf{U}) = -\nabla\cdot\left\langle \int_{-h}^{\zeta} \mathbf{u}'\mathbf{u}' + p\mathbf{I}\, dz \right\rangle + \langle p(z = -h)\rangle\nabla h.$$

An estimate of the bottom pressure term is made by averaging the vertical momentum equation to get

$$\langle p(z = -h)\rangle - g(\bar{\zeta} + h) = \nabla\cdot\left\langle \int_{-h}^{\zeta} w\mathbf{u}\, dz \right\rangle + \left\langle \int_{-h}^{\zeta} w\, dz \right\rangle_t. \quad (11)$$

For slowly-varying small-amplitude waves, the integral terms on the right-hand side may be neglected, and so $\langle p(z = -h)\rangle \approx g(\bar{\zeta} + h)$. Using this in the averaged horizontal momentum equation, and replacing the pressure p with the dynamic pressure $q = p + (z - \bar{\zeta})$ yields

$$(H\mathbf{U})_t + \nabla\cdot(H\mathbf{U}\mathbf{U}) = -\nabla\cdot\mathbf{S} - gH\nabla\bar{\zeta} \quad (12)$$

$$\text{where} \quad \mathbf{S} = \left\langle \int_{-h}^{\zeta} [\mathbf{u}\mathbf{u} + q\mathbf{I}]dz \right\rangle - \left\langle \frac{g}{2}\tilde{\zeta}^2 \right\rangle\mathbf{I}. \quad (13)$$

Here $\mathbf{S}$ is the radiation stress tensor. In the absence of any basic background current, so that $\mathbf{U}$ is $O(a^2)$, we may use the linearized theory (2) to find that

$$\mathbf{S} \approx \mathbf{c}_g\mathbf{k}\frac{E}{\omega} + E\left[\frac{c_g}{c} - \frac{1}{2}\right]\mathbf{I}. \quad (14)$$

where the phase speed $c = \omega/\kappa$, correct to second order in the wave amplitude.

In summary, to this stage the wave field is described by Eqs. (5, 6) for $\mathbf{k}, E$, while the mean field equations to be solved for the mean variables $U, \bar{\zeta}, H$ are the averaged equation for conservation of mass (7) and the averaged equation for conservation of horizontal momentum (12), where the radiation stress tensor is given by (14). An additional equation is needed, and this is provided by the sediment transport flux law described in the next Section 2.3.

2.3. Sediment flux law

To take account of bottom sediment transport, in addition to the kinematic bottom boundary condition,

$$h_t + \mathbf{u}\cdot\nabla h = -w, \quad \text{at} \quad z = -h(\mathbf{x}, t), \quad (15)$$

a second bottom boundary condition is needed, which is an appropriate sediment flux law

$$h_t = \nabla\cdot\mathbf{Q}, \quad (16)$$

where $\mathbf{Q}$ is the sediment flux, evaluated at the bottom. The kinematic condition (15) has already been used in deriving the mean mass Eq. (7). Hence we now also average the sediment flux Eq. (16) so that