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A dynamic, embedded Lagrangian model for ocean climate models. Part I: Theory and implementation

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ABSTRACT

A framework for embedding a Lagrangian model within ocean climate models that employ horizontal Eulerian grids is presented. The embedded Lagrangian model can be used to explicitly represent processes that are at the subgrid scale to the Eulerian model. The framework is applied to open ocean deep convection and gravity driven downslope flows, both of which are subgridscale in the present generation of level coordinate ocean climate models. In order to apply the embedded Lagrangian framework to these processes, it is necessary to partition the mass and momentum of the model into an Eulerian system and a Lagrangian system. This partitioning allows the Lagrangian model to transport seawater using a more appropriate set of dynamics.

A number of schemes suitable for implementation in the embedded Lagrangian model are derived. Two dynamically passive schemes are derived that emulate existing parameterisations and two dynamically active schemes are also derived that evolve Lagrangian parcels of water (termed “blobs”) according to a set of physical equations. Some details of the implementation into the Geophysical Fluid Dynamics Modular Ocean Model are also given. Finally, results are presented that show that the dynamically passive schemes are able to emulate their Eulerian counterparts to within roundoff error in idealised test cases.

1. Introduction

For many applications of computational fluid dynamics, the greatest limiting factor is computational resources. The limitation of computational resources and the very large scale of the domain in global ocean climate modelling means that a number of climatically important processes are unresolved in the present generation of global scale climate models. There has thus been significant effort in the ocean model development community to formulate subgridscale (SGS) parameterisations for all such processes (e.g. see the recent review by Griffies et al. (2010)).

All global scale ocean models that are used for realistic coupled climate experiments utilise a fixed Eulerian grid in the horizontal. There are some models under development that are fully Lagrangian (e.g. Haertel and Randall, 2002), and there are others that have an adaptive mesh (e.g. Piggott et al., 2007), however, none of these other formulations have yet reached a stage where they are able to be used for realistic, coupled global scale studies. Lagrangian coordinates in the vertical (for example, isopycnal coordinate models) are finding much utility in ocean circulation studies (Bleck and Boudra, 1986; Meghann et al., 2010; Dunne et al., 2012). However, the most common class of ocean model used in realistic global scale studies remains the “level” coordinate model (which we take here to mean geopotential, z^*, pressure and p^* coordinates). Henceforth, when we say “Eulerian model” we are referring to an ocean model that solves the primitive equations and has a horizontal grid that is fixed in time.

The fundamental equations of numerical ocean models are typically applied throughout the computational domain, with SGS parameterisations augmenting those equations. For global scale climate models the fundamental equations are the hydrostatic primitive equations. One variation on this idea is the so-called super-parameterisation, first applied in the oceanographic context by Campin et al. (2011). The super-parameterisation is a two dimensional non-hydrostatic model that is embedded into a hydrostatic model. The admission of non-hydrostatic dynamics is fundamentally different to the (hydrostatic) primitive equations of the main model and is well suited for the study of regions of deep convection. Another proposal by Duan et al. (2010) has been to use spatio-temporal filtering of the fully non-hydrostatic Boussinesq equations to better represent vertical motions without the computational cost of using the fully non-hydrostatic equations.

In this paper we propose a rather different means by which to admit dynamics of unresolved or poorly represented physics in

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ocean climate models. To overcome the problems associated with applying a single dynamic regime over the entire computational domain, the mass of the system is partitioned and certain parts of the domain are treated with a different set of dynamics to the bulk of the system. Specifically, we provide a framework in which a Lagrangian model is embedded within the main Eulerian model. The Lagrangian parcels (which we shall refer to as "blobs") generally follow a different set of dynamics to that of the main model. The proposed framework most resembles the cloud-in-cell method (Christiansen, 1973; Mohammadian and Marshall, 2010), however it is not restricted to two dimensional vorticity as is the case in the traditional cloud-in-cell methods. The iceberg model of Martin and Adcroft (2010) is a precedent for our proposed framework within the global scale ocean modelling community. The present framework generalises the ideas behind the iceberg model to be fully three dimensional and fully interactive with the Eulerian model.

Open ocean deep convection and gravity driven downslope flows are our main focus for applying the proposed framework. Both processes are essentially sinking plumes where a relatively narrow water mass is intruding into a much larger water mass. The blobs can thus be considered a Lagrangian discretisation of a plume (or group of plumes). Additionally, blobs can be formed when a certain condition is satisfied, meaning that if the condition is not met, a blob is not formed. This is particularly useful in the context of ocean convection, which is sporadic in both space and time.

This paper presents details of the theory and implementation of the embedded Lagrangian model with application to modelling open ocean deep convection and gravity driven downslope flows. A companion paper, Bates et al. (2012), hereafter BGE-II details the results of tests carried out in model configurations employing idealised bathymetry. This paper consists of the following sections:

- Section 2 outlines the requirements for an Eulerian model to admit an embedded Lagrangian model. Furthermore, it discusses the nature of the Lagrangian parcels and derives tracer mass, seawater mass and momentum budgets for the combined Eulerian and Lagrangian system. The mechanical energy budget is derived in Appendix B.
- Section 3 gives details of the implementation in the Modular Ocean Model (MOM; Bates et al. (2012)), including algorithms for evolving blob properties in time. The aspects of the Eulerian model that must be altered in order to admit the embedded Lagrangian model are also discussed. This section also details some simplifications to the momentum budget of the combined Eulerian and Lagrangian systems.
- Section 4 derives specific equations for open ocean deep convection and gravity currents to illustrate an application of the framework to primitive equation level models. Some "dynamically passive" schemes are presented, the aim of which is to emulate selected parameterisations that are already in existence in purely Eulerian models. Results from these schemes are briefly presented. Other "dynamically active" schemes are also presented, where the equations are derived and discussed.
- Section 5 is a discussion and summary of the theory and background presented, along with prospects for future research and applications.
- Appendix A includes a list of common symbols used throughout this paper.
- Appendix B derives the mechanical energy budget for the combined Eulerian and Lagrangian systems.
- Appendix C derives the blob size required by the NCon-like scheme (see Section 4.3.1).
- Appendix D derives expressions for grid cell variables that are for the combined Eulerian and Lagrangian systems. These are, the total thickness of a grid cell, the total tracer concentration and the total density.

2. Formulation

The purpose of this section is to outline the framework required to embed a Lagrangian model within an Eulerian model when the application is to represent open ocean deep convection and gravity driven downslope flows. Specifically, we discuss implications for the budgets of tracer mass, seawater mass, momentum and mechanical energy.

Large scale ocean models such as MOM are hydrostatic, and there is no overflow parameterisation for hydrostatic models that conserves momentum in the non-hydrostatic sense. Indeed, most existing overflow parameterisations modify only the tracer equation, as the key aim of such parameterisations is to improve water-mass properties as well as transport and formation/conversion rates that are impacted directly by water-masses (e.g. Kösters et al., 2005; Wu et al., 2007; Danabasoglu et al., 2010; Briegleb et al., 2010). In contrast to other schemes, the method proposed in the present paper considers a three-dimensional momentum equation for the Lagrangian blobs, with the vertical momentum equation including acceleration terms absent under the hydrostatic approximation. These vertical accelerations are essential for the vertical movement of the blobs downslope or within a convective plume. However, we are not introducing a fully non-hydrostatic momentum budget for the Lagrangian submodel, since we base the pressure gradients acting on the blobs on the hydrostatic pressure obtained from the Eulerian parent model. Furthermore, by introducing a vertical acceleration to the Lagrangian sub-model, the sum of the Eulerian plus Lagrangian models no longer conserves momentum for the full system. This property is a consequence of allowing for the sub-model to be governed by more complete dynamics than the parent model. This is a property shared by the super-parameterisation scheme of Campin et al. (2011). Nonetheless, we insist on the conservation of scalar properties for the combined model. Conservation of mass, tracer, and heat is a consequence of the partitioning between the E and L systems used in our formulation, and such conservation is an essential property of any parameterisation employed by long-term ocean simulations such as those considered by climate models.

The embedding of a Lagrangian model within an Eulerian model is distinct from the Arbitrary Lagrangian–Eulerian approach (Hirt et al., 1972), in which a mesh may be fixed in the Eulerian sense or move in the Lagrangian sense. In our approach, the Eulerian model remains a true Eulerian model in the sense that it has a fixed mesh and prognostically solves for velocity and tracer concentration at a fixed point in space. The Lagrangian model is a true Lagrangian model in that is solves prognostically for the position and seawater mass and tracer content of a water parcel. To embed the Lagrangian model there must be a rigorous accounting processes in place to ensure that the properties of the combined system remain conservative. We therefore spend a significant amount of this paper deriving budgets for the individual and combined systems.

In order to admit the embedded Lagrangian model the Eulerian model must have, as part of its formulation:

1. Source terms for seawater mass, tracer mass, and momentum,
2. A free surface.

As shall be seen, without source terms in the Eulerian model's formulation, there is no way to transfer seawater mass and tracer mass between the Eulerian model and the Lagrangian model. Without a free surface, there is no way for the Lagrangian blobs to move freely throughout the model domain, and, for the Eulerian model to respond to that movement of seawater mass. The rigid lid approximation (Bryan, 1969) dictates that the volume of water in a given water column must be conserved, thus, to move a blob from