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The impact of mesh adaptivity on the gravity current front speed in a two-dimensional lock-exchange

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A B S T R A C T

Numerical simulations of the two-dimensional lock-exchange flow are used to evaluate the performance of adaptive meshes as implemented in the non-hydrostatic, finite-element model Fluidity-ICOM. The lock-exchange is a widely studied laboratory-scale set-up that produces two horizontally propagating gravity currents and incorporates key physical processes associated with gravity currents over many scales, including ocean overflows. The Froude number (non-dimensional front speed) is used to assess simulations performed on structured-fixed, unstructured-fixed and unstructured-adaptive meshes and different adaptive mesh configurations are compared.

Fluidity-ICOM successfully captures the flow dynamics, including the development of Kelvin–Helmholtz billows. Mesh adapts are guided by a metric which is key to the ability of an adaptive mesh to represent the flow. The metric employed in Fluidity-ICOM is simple, based on the curvature of the solution fields and user-defined solution field weights. Good representation of the gravity current front region is essential to the quality of the solution and for the adaptive meshes this is achieved by reducing the horizontal velocity field weight near the boundaries. Adaptive meshes that are configured in this way are seen to perform as well as high-resolution fixed meshes whilst using at least one order of magnitude fewer nodes. The Froude numbers also compare well with previously published values determined from experimental, numerical and theoretical approaches.

The substantial reduction in the number of nodes used by the adaptive meshes is particularly encouraging as it suggests that even greater gains may be achieved in three-dimensional simulations and larger-scale problems. Results show that successful use of the adaptive mesh approach employed requires a clear understanding of the physics of the system and the metric. These considerations will be vital to the effective application of adaptive mesh approaches in numerical modelling of more complex ocean flows.

1. Introduction

The lock-exchange is a classic laboratory-scale fluid dynamics problem (Fannelop, 1994; Huppert, 2006; Simpson, 1987). A flat-bottomed tank is separated into two sections by a vertical barrier. One section, the 'lock', is filled with the source fluid. This is of different density to the ambient fluid which fills the other section. As the barrier is removed, the denser fluid collapses under the lighter. Two gravity currents form and propagate in opposite directions, one above the other, along the tank. After an initial acceleration, the gravity current fronts travel at a constant speed until the end walls exert an influence or viscous forces begin to dominate (Cantero et al., 2007; Härtel et al., 1999; Huppert and Simpson, 1980). At the current front a bulbous head may develop and become taller than the trailing fluid. Shear instabilities can manifest at the density-interface (hereafter interface) between the two fluids (Turner, 1973), and this leads to the formation of Kelvin–Helmholtz billows that enhance mixing.

This seemingly simple laboratory-scale set-up incorporates the same physical processes encountered in larger scale gravity currents such as sediment-laden density currents and ocean overflows. Sediment-laden density currents can be formed from submarine slides and are known geohazards that have generated tsunamis and caused damage to submarine pipelines and cables (Fine et al., 2005). Ocean overflows, with scales of 10–100 km, funnel and mix dense water between ocean basins and impact upon the meridional overturning circulation (Ivanov et al., 2004; Reid, 1979; Speer and Tziperman, 1990). The lock-exchange idealises these scenarios, presenting a tractable means of studying the processes involved and contributing to our understanding of these real-world flows and their impact.

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The lock-exchange has been the subject of many theoretical, experimental and numerical studies. Theoretical studies generally fall into one of two categories: firstly, an inviscid, hydrostatic current is assumed to be in steady-state and mass and energy balances are calculated across one or both heads (Benjamin, 1968; Shin et al., 2004; von Karman, 1940). This allows the height and speed of the front to be determined but does not account for the influence of the source region. Secondly, an initial value problem is considered which uses the shallow water equations to find the characteristics of the flow (Klemp et al., 1994; Rottman and Simpson, 1983). This allows source effects to be considered but also requires a front condition in order to close the problem. This can be taken from experimental data or steady-state flow theory and if chosen correctly can lead to good agreement between the theory and experimental results (Rottman and Simpson, 1983; Shin et al., 2004). Experimental studies have provided considerable insight into the dynamics of gravity currents including mixing, head dynamics and the Kelvin–Helmholtz instability (Britter and Simpson, 1978; Keulegan, 1958; Simpson and Britter, 1979; Thomas et al., 2003), as well as Reynolds number effects and fixed volume and partial-depth releases (Huppert and Simpson, 1980; Rottman and Simpson, 1983; Shin et al., 2004). Numerical investigations utilise a variety of different discretisation methods, e.g. spectral, finite-element, finite-difference (Elias et al., 2008; Fringer et al., 2006; Härtel et al., 2000; Kao et al., 1978). Such simulations have reproduced the behaviour observed in the laboratory experiments and facilitated investigation of aspects such as the difference between two-dimensional and three-dimensional gravity current flows, e.g. Härtel et al. (2000).

Numerical simulation of gravity currents is challenging. The turbulent dynamics and, in particular, the mixing at the interface between the fluids are governed by non-hydrostatic processes. These are complex and typically small compared to the scale of the whole domain. In real-world scenarios the range of scales involved can be considerable. For example, using a fixed uniform mesh with sufficient resolution to directly resolve the dynamics of an overflow in a ocean basin scale domain will clearly be computationally exorbitant. If the mathematical formulation employed is hydrostatic and/or the resolution is insufficient to capture the turbulent dynamics then a parameterisation is required to represent the mixing in the flow, e.g. Chang et al. (2005), Klemp et al. (1994) and Özgökmen et al. (2007). As a result, overflows are usually represented by a parameterisation in ocean circulation models (Griffies et al., 2001; Haidvogel and Beckmann, 1999; Legg et al., 2006). The promise of adaptive mesh techniques (adaptivity) is that they refine or coarsen the mesh depending on the evolution of flow complexity, allowing the dynamics to be resolved over a range of scales in an efficient manner (Pain et al., 2001; Piggott et al., 2009).

Here, the impact of mesh adaptivity, as implemented in Fluidity-ICOM,[1] on numerical simulations of the two-dimensional lock-exchange problem is evaluated. Fluidity-ICOM is a finite-element model which here solves a non-hydrostatic, Boussinesq formulation of the Navier–Stokes equations, Section 2.1 (Munday et al., 2010; Piggott et al., 2008). The results presented are obtained by numerical simulation on structured-fixed, unstructured-fixed and unstructured-adaptive meshes. Comparison with laboratory experiments and theoretical studies is vital for model assessment. The Froude number (non-dimensional front speed) is often calculated providing an excellent diagnostic for this purpose. With a view to modelling ocean overflows and larger-scale ocean dynamics, the efficient use of computational resources and verification of Fluidity-ICOM is clearly necessary. Furthermore, using an adaptive mesh adds another layer of numerical complexity to the simulation configuration. Therefore, a careful understanding of the impact of adaptivity on the simulated phenomena is key to separating the consequences of using an adaptive mesh from the effects of changing other numerical settings or the physical properties of the problem.

The paper is organised as follows: Sections 2 and 3 describe the physical lock-exchange set-up and Fluidity-ICOM. Section 4 presents and discusses the results from the numerical simulations, comparing them to each other and previously published results. Finally, Section 5 closes with the key conclusions of this work.

[1] http://amcg.ese.ic.ac.uk/FLUIDITY

2. Physical set-up

2.1. Governing equations and parameters

The flow is governed by the Navier–Stokes equations under the Boussinesq approximation, where the density is considered constant except in the buoyancy term:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p - \frac{\rho}{\rho_0} g\mathbf{k} + \nu\nabla^2\mathbf{u}, \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0, \tag{2}$$

with t the time, $\mathbf{u} = (u, v, w)^T$ the velocity field, p the pressure, ρ the density, ρ_0 the background density, g the acceleration due to gravity, ν the kinematic viscosity and $\mathbf{k} = (0,0,1)^T$. These are combined with a linear equation of state and the thermal advection–diffusion equation:

$$\rho = \rho_0 + \rho' = \rho_0(1 - \alpha T), \tag{3}$$

$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T = \kappa\nabla^2 T, \tag{4}$$

with T the temperature, κ the thermal diffusivity and α the thermal expansion coefficient. The values for g, ν, κ, and α are given in Table 1, following primarily the values of Härtel et al. (2000). When Eq. (3) is substituted into Eq. (1), ρ_0 cancels and therefore no value need be set. As the overall density perturbation is at least three orders of magnitude smaller than the background density the use of the Boussinesq approximation is valid (Spiegel and Veronis, 1960).

2.2. The domain, boundary conditions and initial conditions

The domain is a two-dimensional rectangular box, $0 \leqslant x \leqslant 0.8$ m, $0 \leqslant z \leqslant H$, $H = 0.1$ m. As the configuration of the model used here is two-dimensional, motion in the cross-stream (y) direction is neglected. Initially, dense, cold water fills one half of the domain, $T = -0.5$ °C for $x < 0.4$ m, and light, warm water fills the other half, $T = 0.5$ °C for $x \geqslant 0.4$ m, Fig. 4. At $t = 0$ s, $\mathbf{u} = \mathbf{0}$ ms^{-1} everywhere.

A free-slip condition (no normal flow), is applied to the end walls with $u = 0$ ms^{-1} at $x = 0.0$, 0.8 m. A no-slip condition (no flow), is applied at the bottom boundary, $\mathbf{u} = 0$ ms^{-1} at $z = 0$ m

Table 1
Physical parameters for the lock-exchange set-up.

Gravitational acceleration (ms^{-2})	g	10
Kinematic viscosity (m^2 s^{-1})	ν	10^{-6}
Thermal diffusivity (m^2 s^{-1})	κ	0
Thermal expansion coefficient (°C^{-1})	α	10^{-3}
Domain height (m)	$H = 2h$	0.1
Reduced gravity (ms^{-2})	$g' = g\frac{\rho_1-\rho_2}{\rho_0} = -g\alpha(T_1 - T_2)$	10^{-2}
Buoyancy velocity (ms^{-1})	$u_b = \sqrt{g'H}$	$\sqrt{10^{-3}}$
Grashof number	$Gr = \left(\frac{h\sqrt{g'h}}{\nu}\right)^2$	1.25×10^6