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Energy transfers in internal tide generation, propagation and dissipation in the deep ocean

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ABSTRACT

The energy transfers associated with internal tide (IT) generation by a semi-diurnal surface tidal wave impinging on a supercritical meridionally uniform deep ocean ridge on the *f*-plane, and subsequent IT-propagation are analysed using the Boussinesq, free-surface, terrain-following ocean model *Symphonie*. The energy diagnostics are explicitly based on the numerical formulation of the governing equations, permitting a globally conservative, high-precision analysis of all physical and numerical/artificial energy transfers in a sub-domain with open lateral boundaries. The net primary energy balances are quantified using a moving average of length two tidal periods in a simplified control simulation using a single time-step, minimal diffusion, and a no-slip sea floor. This provides the basis for analysis of enhanced vertical and horizontal diffusion and a free-slip bottom boundary condition. After a four tidal period spin-up, the tidally averaged (net) primary energy balance in the generation region, extending ±20 km from the ridge crest, shows that the surface tidal wave loses approximately $C = 720$ W/m or 0.3% of the mean surface tidal energy flux (2.506×10^5 W/m) in traversing the ridge. This corresponds mainly to the barotropic-to-baroclinic energy conversion due to stratified flow interaction with sloping topography. Combined with a normalised net advective flux of baroclinic potential energy of $0.9 \times C$ this causes a net local baroclinic potential energy gain of $0.72 \times C$ and a conversion into baroclinic kinetic energy through the baroclinic buoyancy term of $1.18 \times C$. Tidally averaged, about $1.14 \times C$ is radiated into the abyssal ocean through the total baroclinic flux of internal pressure associated with the IT- and background density field. This total baroclinic pressure flux is therefore not only determined by the classic linear surface-to-internal tide conversion, but also by the net advection of baroclinic (background) potential energy, indicating the importance of local processes other than linear IT-motion. In the propagation region (PR), integrated over the areas between 20 and 40 km from the ridge crest, the barotropic and baroclinic tide are decoupled. The net incoming total baroclinic pressure flux is balanced by local potential energy gain and outward baroclinic flux of potential energy associated with the total baroclinic density. The primary net energy balances are robust to changes in the vertical diffusion coefficient, whereas relatively weak horizontal diffusion significantly reduces the outward IT energy flux. Diapycnal mixing due to vertical diffusion causes an available potential energy loss of about 1% of the total domain-averaged potential energy gain, which matches $\frac{k_m-1}{k_m}\rho_0 K^V N^2$ to within 0.5%, for k_m linearly distributed grid-levels and constant background density ρ_0, vertical diffusivity (K^V) and buoyancy frequency (N).

1. Introduction

The world's oceans dissipate about 3.6 TW of the tidal energy in the earth–moon–sun system, of which 2.54 TW is associated with the semi-diurnal (M2) surface tide (Cartwright and Ray, 1991; Egbert and Ray, 2001). By interaction with sloping sea floors, roughly 1 TW of M2 surface tidal energy is converted into M2 internal tides (Egbert and Ray, 2001; Wunsch and Ferrari, 2004), which are observed to propagate for thousands of kilometres from their generation regions, such as the Hawaiian ridge (Ray and Mitchum, 1997). Thus, internal tides (IT) appear to play a fundamental role in the meridional overturning circulation, providing the abyssal ocean with half of the small-scale mixing energy required to maintain the observed global stratification (Munk and Wunsch, 1998; Wunsch and Ferrari, 2004). The required energy cascade from large to small-scale internal waves may occur through the formation of higher harmonics due to wave-wave interactions, parametric sub-harmonic instability (Gerkema et al., 2006a), reflection of IT beams (Gerkema et al., 2006b) and fission of low-mode internal waves into solitons (e.g. Gerkema and Zimmerman, 1995; Shaw et al., 2009). Small-scale internal wave breaking, resulting from a multitude of processes reviewed by

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Thorpe (2005), causes overturning of isopycnal surfaces and irreversible diapycnal transfer of heat and salinity.

Even with the advent of dedicated large-scale campaigns (Lien and Gregg, 2001; Rudnick et al., 2003), existing *in situ* measurements cannot cover the IT energy transfers through the wide range from basin scale tidal dynamics to millimetric mixing. Altimetry and SAR observations provide ample data on propagation of low-mode internal tides and solitons, but cannot tell us much about weaker, high-mode internal tide motion (Egbert and Ray, 2001; Apel, 1987; Hyder et al., 2005; Niwa and Hibiya, 2001).

Time-averaged tidal barotropic-to-baroclinic energy conversion is often assumed to match the local internal tide energy flux divergence, which is modelled as

$$\left\langle \vec{\nabla} \cdot (\vec{v}'p') \right\rangle_T = -\rho_0 \langle b' \bar{v}_z \rangle_T, \tag{1.1}$$

where $\vec{v}'$, p', and b' are the baroclinic velocity, pressure and buoyancy anomaly and $\bar{v}_z$ the vertical velocity induced by barotropic tidal flow over sloping topography, while ρ_0 is a constant background density and $\langle \cdot \rangle_T$ indicates the temporal average (Niwa and Hibiya, 2001; Holloway and Merrifield, 1999; Gerkema et al., 2004; Di Lorenzo et al., 2006). This formulation is inappropriate for large-amplitude internal tides, which would require inclusion of (non-linear) advective energy fluxes (Scotti et al., 2006; Lamb, 2007) and does not explicitly represent the energy lost by the surface tidal wave.

Recent theoretical estimates are constrained to two-dimensional motion, the rigid-lid or infinite-depth assumptions, inviscid, non-diffusive and (weakly non-) linear cases (Baines, 1982; Bühler and Muller, 2007; Pétrélis et al., 2006; Khatiwala, 2003; Llewellyn Smith and Young, 2002; Lamb, 2007). Viscosity is essential to internal tide theory, which otherwise contains small-scale singularities in the internal tide beams (Pétrélis et al., 2006). Peacock et al. (2008) showed good correspondence between laboratory experiments and weakly viscous, linear theory, but were limited to either sub-critical or knife-edge topography of small height compared to the fluid depth. Non-linearity, viscosity and inclusion of a free surface to explicitly calculate tidal energy conversion render the theory intractable.

A global-scale model that accurately represents fine-scale mixing is not currently within reach, although low-resolution global simulations point to primary IT-production and conversion sites (Simmons, 2008). Recent regional-scale efforts focused on M2-tidal conversion and internal tide energy flux (Holloway and Merrifield, 1999; Munroe and Lamb, 2005; Di Lorenzo et al., 2006; Katsumata, 2006; Carter et al., 2008). Carter et al. (2008) used POM to estimate the M2 internal tide energetics around the Hawaiian ridge, employing the sum of kinetic and linearised available potential energy, which is appropriate for linear stratification and small-amplitude internal waves, discretised on the horizontal and vertical mid-point of the model grid-cells. Although they concluded that local dissipation is a non-negligible factor in the IT generation zone, they observed an error in both the barotropic and baroclinic global energy balance on the order of 10% of the primary energy conversions.

We propose a complete energetics analysis of the internal tide from generation to mixing, explicitly based on the numerical scheme of the energy-conserving, Boussinesq, hydrostatic, free-surface, terrain-following (σ-) coordinate ocean model *Symphonie*, which is similar to POM and described in detail by Marsaleix et al. (2008). We evaluate *all* physical and numerical energy transfers, permitting quantification, control and motivation of the artificial energy transfers due to model choices, for instance temporal diffusion and discretisation onto the C-grid (Arakawa and Lamb, 1977). Expanding upon the global formulation for closed basins by Marsaleix et al. (2008), we also consider lateral boundary fluxes in sub-domains, e.g. to estimate internal tide energy radiation from the generation region.

We analyse M2 internal tide generation by a barotropic surface wave that impinges on a supercritical Gaussian ridge in the rotating stratified deep ocean, a representative case related to the Hawaiian Island chain, previously studied in detail by Holloway and Merrifield (1999), Munroe and Lamb (2005) and Lamb (2007). We aim to quantify numerically (a) the primary energy balances in internal tide generation and in IT propagation away from topography and (b) the associated energy expended in diapycnal diffusion. The energy lost to enhanced diapycnal mixing associated with the internal tide is evaluated following Winters et al. (1995), using a novel adiabatic redistribution algorithm adapted to terrain-following coordinates and a free surface. Finally, we underline the importance of respecting the discretised model formulation for precise energy diagnostics.

The paper is organised as follows: In Section 2, the model setup is introduced and following parameter space analysis the resulting internal tide field is characterised. In Section 3, the energy evolution equations are presented, which are adapted to internal tide generation in Section 4. In Section 5, we show the energy balance is closed, analyse the numerical energy transfers and the primary energy transfers in internal tide generation and propagation and evaluate the effects of enhanced vertical and horizontal diffusion and a free-slip bottom boundary condition. We discuss the results and conclude and present perspectives in Section 6.

2. Numerical simulation of the internal tide

The energy transfers inherent in internal tide generation by free-surface tidal flow over a Gaussian ridge are studied using the hydrostatic approximation, on the traditional *f*-plane. The model equations are presented in Appendix A.1 and discretised on the C-grid in the horizontal and on the Lorenz grid in the vertical, using second order accurate schemes for advection of momentum and tracers and an explicit leapfrog scheme in time, using a single time-step. Vertical diffusion is calculated using an implicit scheme. A Robert-Asselin filter is applied to the momentum and tracer equations to limit high-frequency numerical noise associated with the leapfrog scheme (Robert, 1966; Asselin, 1972; Marsaleix et al., 2008, Section 3.2.2). A control simulation is defined in Section 2.1, aimed primarily at the analysis of mechanical energy conversion from the barotropic to the baroclinic tide. Therefore, numerical and physical viscosity and tracer diffusivity are minimised, while maintaining model stability.

2.1. Model setup

The numerical model is set up in the zonal/vertical *Oxz*-plane and is uniform and cyclic in the *Oy*-direction, permitting the Coriolis effect (cf. Appendix A.2), with $f = 10^{-4}\ \mathrm{s}^{-1}$, corresponding to a latitude of 43.4°N. The numerical domain extends 1200 km zonally, with ambient depth H = 5 km and a Gaussian ridge defined by

$$h = h_0 \exp\left\{-\frac{(x - x_0)^2}{a^2}\right\}, \tag{2.1}$$

centred at x_0 = 600 km from the western boundary, and of characteristic height h_0 = 1500 m and width a = 6.45 km. The horizontal resolution is Δx = 1 km and in the vertical 40 linearly distributed σ-layers are used.

Initially, the fluid is salt-stratified with a constant buoyancy frequency $N = 10^{-3}\ \mathrm{s}^{-1}$. With the linear equation of state (A5), the heat and salinity Eqs. (A4a) and (A4b) reduce to a single evolution equation for the density anomaly