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A conservative nonlinear filter for the high-frequency range of wind wave spectra ☆

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ABSTRACT

Filtering of the high-frequency part of a wind wave spectrum may be useful in a numerical wind wave model for various reasons. First, it can be used to augment (or be part of) a parameterization of the resonant nonlinear interactions, that are essential to third-generation wind wave models. Second, when combined with a dynamic time stepping scheme for source term integration, it may result in smoother (and hence faster) wave model integration. In this study, such a filter is proposed, based on the traditional Discrete Interaction Approximation (DIA) for the resonant four-wave nonlinear interactions. This filter retains all conservative properties of the interactions. For small time steps and/or smooth spectra, it is formulated as a traditional source term. For larger time steps and/or non-smooth spectra it is formulated as a filter. This formulation guarantees stability of the filter itself and will enhance overall computational stability in a full wave model. The stability properties of this filter are illustrated using traditional wave growth computations. Examples are given where the filter improves model economy, and where it is shown to remove spurious high-frequency noise from a wave model.

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1. Introduction

Wind waves at sea represent a stochastic process. The characteristics of such waves are generally described using wave energy or variance spectra, based on the original work with radio waves of Rice (1944). Typically such a spectrum is described in terms of frequencies f and directions θ associated with spectral wave components. Away from the surf zone, the phase information of spectral components is generally ignored resulting in a random-phase approach, and only the spectral energy level is considered. The evolution of the spectral energy is described or modeled using a spectral balance equation (Hasselmann, 1960), which in its simplest form becomes

$$\frac{DF(f,\theta)}{Dt} = S_{in}(f,\theta) + S_{nl}(f,\theta) + S_{ds}(f,\theta), \qquad (1)$$

where $F(f,\theta)$ is the energy or variance spectrum, S_{in} is a source term describing wind-wave interactions (wind input), S_{nl} describes nonlinear interactions between wave components in the spectrum, and S_{ds} describes wave dissipation, typically associated with wave breaking or 'whitecapping'. The left side of this equation describes (conservative) linear wave propagation.

The nonlinear interactions S_{nl} describe third-order resonant nonlinear interactions between four wave components. It is critical in describing wave growth, as these resonant interactions are generally believed to be the lowest order processes able to shift energy to lower frequencies (longer waves) at arbitrary depth, and to result in uniform spectral shapes at high frequencies (e.g., Hasselmann et al., 1973). In third-generation wave models, defined as models where spectral shapes are determined by the explicit source term balance, rather than by resorting to pre-described spectral shapes and energy levels, the parameterization of nonlinear interactions is of paramount importance. Such a third-generation approach to wind wave modeling is believed to be essential for accurate general-purpose wind wave models (SWAMP group, 1985).

Explicitly computing (the balance of) all source terms up to high frequencies introduces complications in numerical wind wave models. First, time scales of spectral change at high frequencies become small, resulting in prohibitively small numerical time steps if the time evolution of the corresponding spectral components is to be resolved. In many operational wave models a simple solution to avoid computations with excessively small time scales is to apply a parametric spectral shape ("tail") to frequencies above typically 3 times the spectral peak frequency (f_p) as associated with a growing wind sea. However, small time scales also occur in the frequencies $f_p < f < 3f_p$. Fortunately, source terms in this frequency ranges generally are in near-equilibrium conditions. This property is explicitly used in common time integration schemes for source terms in third-generations wave models (e.g., WAMDIG, 1988; Tolman, 1992; Hargreaves and Annan, 1998, 2001), allowing for economically acceptable numerical time steps. Another numerical acceleration technique is based on the introduction of so-called limiters, restricting the discrete change of energy per time step

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per discrete spectral bin. Note that some limiters result in non-convergent model behavior (e.g., Hersbach and Janssen, 1999; Hersbach and Janssen, 2001; Tolman, 2002), and are therefore economically useful, but numerically suspect. Such limiters operate on individual spectral bins, and hence are non-conservative by nature.

Second, full computations of S_{nl} are prohibitively expensive for operational wave models, and economical yet accurate parameterizations have been elusive (Section 2). At least part of the problem with developing economical parameterizations of S_{nl} is that both large and small spectral-space scales of the exact interactions are important in describing wave growth. Interactions at larger spectral scales enable the (slow) evolution of the spectrum toward longer and higher waves, whereas small scale interactions are needed to stabilize the spectral shape at higher frequencies toward the local equilibrium solution (e.g., Young and Van Vledder, 1993). Many parameterizations have been suggested based on the large scale features of the nonlinear interactions, but almost without exceptions such parameterizations do not result in viable numerical models since the small-scale interactions are not represented adequately. Only a highly simplified discrete analog to the full interactions (DIA, Hasselmann et al., 1985) has been proven adequate for use in numerical models. After 25 years, the DIA with minor modifications is still the only interaction parameterization used in operational wave models. The only approach that could be considered an exception to this is the SRIAM method used in Japan (e.g., Tamura et al., 2008).

Both issues with numerical model integration at high frequencies are related to obtaining a near-equilibrium spectral solution at frequencies above the spectral peak frequency with numerical time steps that are larger than the evolutionary time scales involved. Ideally this can be achieved with a robust integration scheme designed to find near-equilibrium solutions (e.g., Hargreaves and Annan, 1998; Hargreaves and Annan, 2001), combined with a parameterization of S_{nl} that adequately describes the return to equilibrium solutions of a perturbed spectrum, as described in, e.g., Young and Van Vledder (1993). Present third generation models provide such an environment using the DIA, in spite of its many shortcomings (e.g., Van Vledder et al., 2000). Other proposed parameterizations of S_{nl}, however, are only partially successful in creating a dynamic source term balance at higher frequencies. Models using such parameterizations might benefit from filtering techniques that remove high-frequency noise from the resulting spectra. Due to the dominant role of the nonlinear interactions in stabilizing spectral shapes for frequencies above the spectral peak, it appears prudent for such a filter to have conservation and equilibrium properties of the nonlinear interactions. Such a filter is developed in the present study. It is shown that the traditional DIA can be reduced to a local quasi-diffusion. This reduced version of the DIA can be used as a source term that converts to a filter for large (economically feasible) time steps. It effectively adds a separate DIA configuration to a limited part of the spectral space, and is applied outside the general source term integration of a model. It is shown to reduce noise and accelerate model integration in a model using a Neural Network approximation to the interactions, and is shown to remove spurious high-frequency noise and increase model accuracy in some Generalized Multiple DIA configurations used to model nonlinear interactions.

Section 2 discusses nonlinear interactions and their parameterizations as relevant for the present study. In Section 3 the source term and filter is derived, and its parameter settings are assessed in the context of full wave growth in Section 4. In Section 5 the filter is applied to several practical wave modeling problems to show its potential impact. A discussion and conclusions are presented in Section 6.

2. Nonlinear interactions

The exact computation of the nonlinear interactions S_{nl} involves the evaluation of a six-dimensional Boltzmann integral. It includes an interaction function with strong moving singularities (e.g., Webb, 1978; Herterich and Hasselmann, 1980), and delta functions with contributions only for resonant sets of four wave components (so-called quadruplets), satisfying (Hasselmann, 1962, 1963)

$$\boldsymbol{k}_1 + \boldsymbol{k}_2 = \boldsymbol{k}_3 + \boldsymbol{k}_4, \tag{2}$$

$$\sigma_1 + \sigma_2 = \sigma_3 + \sigma_4. \tag{3}$$

where $\boldsymbol{k}$ represent wavenumber vectors, and σ represents the corresponding intrinsic frequencies, related in the (deep water) dispersion relation

$$\sigma^2 = gk. \tag{4}$$

This effectively reduces the integral to a three-dimensional integral over spectral space. Even with present day computers, and with various improvements in the efficiency of the computation of these integrals (e.g., Masuda, 1980; Tracy and Resio, 1982; Resio and Perrie, 1991; Komatsu and Masuda, 1996; Van Vledder, 2000; Van Vledder, 2006) the exact integral is prohibitively expensive for use in practical models. When numerical packages for exact computations (e.g., Van Vledder, 2000; Van Vledder, 2006) are applied in commonly used wave models, they increase model run times typically by up to three orders of magnitude.

So far, only one cheap approximation to the nonlinear interactions has been proven suitable for incorporation in operational wave models. This is the Discrete Interaction Approximation (DIA) of Hasselmann et al. (1985). In this approximation, a single representative resonant quadruplet is considered, satisfying

$$\left.\begin{array}{l} \boldsymbol{k}_2 = \boldsymbol{k}_1 \\ \sigma_3 = (1+\lambda)\sigma_1 \end{array}\right\}, \tag{5}$$

where λ is a constant, typically $\lambda = 0.25$, and where contributions to the interactions for the quadruplet components $\delta S_{nl,i}$ at $\boldsymbol{k}_i$ are computed as

$$\begin{pmatrix} \delta S_{nl,1} \\ \delta S_{nl,3} \\ \delta S_{nl,4} \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} Cg^{-4} f_1^{11} \left[F_1^2 \left(\frac{F_3}{(1+\lambda)^4} + \frac{F_4}{(1-\lambda)^4} \right) - \frac{2F_1F_3F_4}{(1-\lambda^2)^4} \right], \tag{6}$$

where $F_i = F(f_i, \theta_i)$, and C is a constant, typically $C = 3 \times 10^7$. This expression is evaluated for $\boldsymbol{k}_i$ equivalent to each discrete spectral grid point, after which the total interaction is obtained by summation of all discrete contributions δS_{nl}. Note that $\boldsymbol{k}_3$ and $\boldsymbol{k}_4$ generally do not coincide with the discrete spectral grid, so that F_3 and F_4 need to be evaluated by bi-linear interpolation. Likewise, $\delta S_{nl,3}$ and $\delta S_{nl,4}$ need to be distributed over surrounding discrete spectral grid points, consistent with the bi-linear interpolation of F_3 and F_4. Implicit to Eq. (6) is the assumption of a logarithmic discrete frequency grid

$$f_{i+1} = X_f f_i, \tag{7}$$

where typically $X_f = 1.10$ for operational wave models and $X_f = 1.07$ for research models using the exact Boltzmann integral. Eq. (6) is based on a discrete equivalent to the full integration integral, and retains the conservation of energy, action and momentum implicit to the interactions, as long as the quadruplet satisfies the resonance conditions (Webb, 1978). Note that the interpolations of F, and distribution of δS_{nl} retain the conservation of these three quantities (e.g., Tolman, 2008, Section 2.2).

The DIA has been highly successful in making third-generation wave models feasible. However, it has also long been known to