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Comparison of four mixed layer mesoscale parameterizations and the equation for an arbitrary tracer

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ABSTRACT

In this paper we discuss two issues, the inter-comparison of four mixed layer mesoscale parameterizations and the search for the eddy induced velocity for an arbitrary tracer. It must be stressed that our analysis is limited to mixed layer mesoscales since we do not treat sub-mesoscales and small turbulent mixing.

As for the first item, since three of the four parameterizations are expressed in terms of a stream function and a residual flux of the RMT formalism (residual mean theory), while the fourth is expressed in terms of vertical and horizontal fluxes, we needed a formalism to connect the two formulations. The standard RMT representation developed for the deep ocean cannot be extended to the mixed layer since its stream function does not vanish at the ocean's surface.

We develop a new RMT representation that satisfies the surface boundary condition. As for the general form of the eddy induced velocity for an arbitrary tracer, thus far, it has been assumed that there is only the one that originates from the curl of the stream function. This is because it was assumed that the tracer residual flux is purely diffusive.

On the other hand, we show that in the case of an arbitrary tracer, the residual flux has also a skew component that gives rise to an additional bolus velocity. Therefore, instead of only one bolus velocity, there are now two, one coming from the curl of the stream function and other from the skew part of the residual flux. In the buoyancy case, only one bolus velocity contributes to the mean buoyancy equation since the residual flux is indeed only diffusive.

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1. Introduction

In this work we discuss two issues: the intercomparison of four available mixed layer ML mesoscale parameterizations and whether the eddy induced velocity for buoyancy can also represent tracers other than buoyancy, for example, passive tracers such CO_2, CFC, etc., that form part of climate studies. We study mixed layer mesoscales only, with no reference to sub-mesoscales and small scale turbulent mixing which require parameterizations not discussed here.

As for the first item, three of the four parameterizations are expressed in terms of the stream function $\mathbf{\Psi}$ and residual flux $\mathbf{F}_r$ of the residual mean theory RMT formalism (Aiki et al., 2004; Ferrari et al., 2008, 2010, cited as A4, F8, 10), while the fourth one (Canuto et al., submitted for publication, cited as C11) is expressed in terms of the vertical–horizontal mesoscale fluxes $(F_v, \mathbf{F}_H)$.[1] To carry out the model intercomparison, we need to translate the C11 $(F_v, \mathbf{F}_H)$ formulation into the corresponding formulation in terms of $(\mathbf{\Psi}, \mathbf{F}_r)$.

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[1] Killworth (2005, K5) was the first to argue that in the ML flows occur mostly within on horizontal planes and thus the natural representation of the mesoscale tracer flux is in terms of the horizontal $\mathbf{F}_H$ and vertical F_v component. K5 solved the linear mesoscale dynamic equations and showed that F_v is a skew flux, i.e., its divergence yields an horizontal advection with a bolus-like velocity $\mathbf{u}^*$, while $\mathbf{F}_H$ is of the diffusion type with a mesoscale diffusivity κ_M. Given the linear character of the model, K5 was unable to derive the strength of either $\mathbf{u}^*$ and/or of κ_M. Recently, K5's analysis was extended to include the non-linear terms (C11) and the form of $\mathbf{F}_H$ and F_v *for an arbitrary tracer* in terms of the large scale fields was derived. When the tracer was the buoyancy field, the parameterization was assessed in several ways, e.g., z-profile of the eddy kinetic energy vs. WOCE data, surface eddy kinetic energy vs. T/P altimetry data, dependence of the vertical flux on the mean velocity against eddy resolving simulation data, etc. The $(\mathbf{\Psi}, \mathbf{F}_r)$ representation has the advantage of facilitating the matching with the ocean interior at the bottom of the mixed layer while the $(F_v, \mathbf{F}_H)$ representation has a different advantage. Since the dynamic equation for the EKE (eddy kinetic energy, see e.g., Boning and Budich, 1992) shows that F_v acts as a source of EKE, one can model the surface eddy kinetic energy by averaging the vertical buoyancy flux F_v over the mixed layer and then assess the result against the T/P data (Scharffenberg and Stammer, 2010).

Since the standard form of ($\mathbf{\Psi}$, $\mathbf{F}_r$) cannot be used in the ML since it does not satisfy the boundary condition $\mathbf{\Psi}(0) = 0$, we developed a new RMT formulism valid in the ML. The final result, Eq. (21), expresses (F_v, $\mathbf{F}_H$) in terms of ($\mathbf{\Psi}$, $\mathbf{F}_r$).

The results of the four models intercomparison can be summarized as follows: (a) the F8 bolus velocity does not entail ML re-stratification which is known to exist, (b) the A4, F10 bolus velocities induce re-stratification but at the lowest order in the smallness parameter h/H, A4,F10 are not different (h, H are the ML and ocean depths, respectively), (c) using the $\mathbf{\Psi}$ of A4, F8,10, we construct the corresponding $\mathbf{F}_r$'s; we reproduce the F8 result while the $\mathbf{F}_r$ of A4, F10 are new since they were not given in the original work, (d) in both A4, F10, $\mathbf{F}_r$ is diffusive with a mesoscale diffusivity that vanishes at the bottom of the ML, as expected, (e) however, their values at the surface is h/H times smaller than the value obtained by Zhurbas and Oh (2003), and finally, (f) only the C11 model accounts for wind and mean flow, which affect both the mesoscale fluxes and their kinetic energy.

Concerning the parameterization of an arbitrary tracer, we obtain the following results. In addition to a diffusive component, the residual flux exhibits a new feature, a *skew component,* which gives rise to an additional bolus velocity. There are therefore two mesoscale advection terms: one due to the bolus velocity originating from the stream function and the other from the bolus velocity originating from the skew part of the residual flux. The common assumption that there is only one bolus velocity is therefore no longer tenable (in the case of buoyancy, only the bolus velocity from the stream function contributes to the mean buoyancy equation).

2. Inapplicability of the standard RMT to the ML

Consider the model independent dynamical equation for the mean buoyancy $b = -g\rho_0^{-1}\rho$ (e.g., Ferreira et al., 2005, Eq. (1))

$$\partial_t \bar{b} + \overline{\mathbf{U}} \cdot \nabla\overline{\mathbf{b}} + \nabla \cdot \mathbf{F}(\overline{\mathbf{b}}) = -\nabla \cdot \mathbf{F_{SM}}(\overline{\mathbf{b}}) - \partial_z \mathbf{F_{ss}} + \mathbf{Q} \tag{1}$$

Here, $\overline{\mathbf{U}} = (\overline{\mathbf{u}}, \overline{\mathbf{w}})$ is the mean velocity and $\mathbf{F}(b) = \overline{\mathbf{U}'b'}$ is the 3D mesoscale buoyancy flux with horizontal-vertical components $\mathbf{F}_H(b) = \overline{\mathbf{u}'b'}, F_v(b) = \overline{w'b'}$, $\mathbf{U}' = (\mathbf{u}', w')$ is the mesoscale velocity field and ∇ is the 3D nabla operator; the overbar stands for an ensemble average.[2] The first and second terms on the rhs represent the contribution due to sub-mesoscales and small scale (ss) turbulence (which we write for completeness but which we do not treat in this work), while Q stands for sources and sinks. The RMT decomposition of the buoyancy flux $\mathbf{F}(b)$ into isopycnal–diapycnal components is as follows (Andrews and McIntyre, 1976; Treguier et al., 1997; Plumb and Ferrari, 2005; Ferreira et al., 2005; F8):

$$\mathbf{F}(b) \equiv \overline{\mathbf{U}'b'} = \mathbf{\Psi} \times \nabla\bar{b} + \mathbf{F}_r(b) \tag{2}$$

where the *stream function pseudo-vector* $\mathbf{\Psi}$ and the *residual vector flux* $\mathbf{F}_r$ are defined as follows:

$$\mathbf{\Psi} = -\frac{\mathbf{F}(b) \times \nabla\bar{b}}{|\nabla\bar{b}|^2} = \frac{1}{|\nabla\bar{b}|^2}\left[(F_v\nabla_H\bar{b} - N^2\mathbf{F}_H) \times \mathbf{e}_z - \mathbf{F}_H \times \nabla_H\bar{b}\right] \tag{3}$$

[2] The average in the mesoscale flux (1) stems from the non-linear term in the equation for the instantaneous buoyancy field when averaged over a coarse resolution grid cell of horizontal scale ~100 km. However, averaging over the grid cell in (1) is not sufficient in, say, testing a mesoscale flux parameterization using high resolution simulations. The reason is that the diagnosed fluxes are random functions of the large scale fields which are the ones averaged over the grid cell. To obtain deterministic functions from high resolution data, one needs to ensemble average the random functions. If the large scale fields are stationary and/or homogeneous for sufficiently long time and/or within large area, the diagnosed instantaneous fluxes averaged over the grid cell may be further averaged over the corresponding time and/or space intervals instead of ensemble averaging. Below, we imply instantaneous averages over the grid cell together with ensemble averages.

$$\mathbf{F}_r(b) = \frac{\mathbf{F}(b) \cdot \nabla\bar{b}}{|\nabla\bar{b}|^2}\nabla\bar{b} = \frac{\nabla_H\bar{b} + N^2\mathbf{e}_z}{|\nabla\bar{b}|^2}\left[\mathbf{F}_H \cdot \nabla_H\bar{b} + F_v N^2\right] \tag{4}$$

where $\mathbf{e}_z$ is the vertical unit vector and N is the Brunt–Vaisala frequency. The first term in (2) representing the isopycnal component of the buoyancy flux, has the form of a skew flux (Griffies, 1998) and its divergence leads to an advection:

$$\nabla \cdot (\mathbf{\Psi} \times \nabla\bar{b}) = \mathbf{U}^+ \cdot \nabla\bar{b} \tag{5a}$$

where $\mathbf{U}^+$ is the eddy induced or bolus velocity:

$$\mathbf{U}^+ = \nabla \times \mathbf{\Psi}, \quad \nabla_H \cdot \mathbf{u}^+ + \partial_z w^+ = 0 \tag{5b}$$

It follows that the mesoscale buoyancy flux $\mathbf{F}(b)$ contributes to Eq. (1) as an advection and a diffusion:

$$\nabla \cdot \mathbf{F}(b) = \mathbf{U}^+ \cdot \nabla\bar{b} + \nabla \cdot \mathbf{F}_r \tag{5c}$$

In a fully adiabatic ocean, that is, one with no diabatic ML, the residual flux $\mathbf{F}_r$ is negligible and thus the main mesoscale effect is represented by the first, advective, term in (5c). As McDougall and McIntosh (2001) showed, at the ocean's surface the stream function satisfies the boundary condition $\mathbf{\Psi}(0) = 0$. We concentrate on the horizontal component of this condition:

$$\mathbf{\Psi}_H(0) = 0 \tag{6a}$$

since (6a) ensures the vanishing of the vertical component of the eddy induced velocity (5b) at the ocean's surface:

$$w^+(0) = \mathbf{e}_z \cdot [\nabla_H \times \mathbf{\Psi}_H(0)] = 0 \tag{6b}$$

Furthermore, condition (6a) leads to the vanishing of the vertical component of the isopycnal flux at the surface:

$$\mathbf{\Psi}(0) \times \nabla\bar{b} \cdot \mathbf{e}_z = 0 \tag{6c}$$

Since the vertical component of the full flux also vanishes at the surface:

$$\mathbf{F}(0) \cdot \mathbf{e}_z \equiv F_v(0) = 0 \tag{7}$$

an analogous boundary condition must be satisfied by the residual flux:

$$\mathbf{F}_r(0) \cdot \mathbf{e}_z = 0 \tag{8}$$

How does the presence of the diabatic ML affect conditions (6)–(8)? To answer the question, we consider $\mathbf{\Psi}_H$ near surface. Since the last term in (3) does not contribute to $\mathbf{\Psi}_H$ and the first term is very small, strictly, it vanishes at $z = 0$ because of (7), we consider the term:

$$\mathbf{\Psi}_H(z) = -\frac{N^2(z)\mathbf{F}_H(z) \times \mathbf{e}_z}{N^4(z) + |\nabla_H\bar{b}|^2} \tag{9a}$$

Near the surface, the horizontal flux $\mathbf{F}_H$ does not vanish as it follows from the observational result by Zhurbas and Oh (2003):

$$\mathbf{F}_H(0) = -\kappa_s\nabla_H\bar{b}, \quad \kappa_s \equiv \kappa_M(0) = C\ell K^{1/2}(0) \tag{9b}$$

who arrived at it using data from the Global Drifter Program/Surface Velocity Program. In (9b), $\kappa_M(z)$ is the mesoscale diffusivity and $C = 1.02 \pm 0.13$; furthermore $\ell = \min(r_d\, L_R)$ where r_d is the Rossby deformation radius and L_R is the Rhines scale. The surface mesoscale eddy kinetic energy $K(0)$ can be obtained from the T/P data (Scharffenberg and Stammer, 2010). Since in the ocean, even near the surface, one has $s < 1$ (typical isopycnal slopes below the ML are of the order of 10^{-3} while in the ML they are about an order of magnitude larger) from (9a,b) we conclude that condition (6a) and therefore (6b,c), are not satisfied. In addition, the interpretation of the divergence of the skew flux (5a) as an advection becomes problematic. In fact, in the limit $s \ll 1$, from (9a,b) we obtain that the vertical bolus velocity becomes: