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Ocean turbulence, III: New GISS vertical mixing scheme

V.M. Canuto [a,b,*], A.M. Howard [a,c], Y. Cheng [a,d], C.J. Muller [e], A. Leboissetier [a,e], S.R. Jayne [f]

[a] NASA, Goddard Institute for Space Studies, New York, NY, 10025, USA
[b] Department of Applied Physics and Applied Mathematics, Columbia University, New York, NY, 10027, USA
[c] Department of Physical Environmental and Computer Science, Medgar Evers College, CUNY, New York, NY, 11225, USA
[d] Center for Climate Systems Research, Columbia University, New York, NY, 10025, USA
[e] Department of Earth, Atmospheric and Planetary Sciences, MIT, Cambridge, MA, 02138, USA
[f] Physical Oceanography Department, WHOI, Woods Hole, MA, 02543, USA

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The authors dedicate this work to the memory of Peter Killworth.

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ABSTRACT

We have found a new way to express the solutions of the RSM (Reynolds Stress Model) equations that allows us to present the turbulent diffusivities for heat, salt and momentum in a way that is considerably simpler and thus easier to implement than in previous work. The RSM provides the dimensionless *mixing efficiencies* Γ_α (α stands for heat, salt and momentum). However, to compute the diffusivities, one needs additional information, specifically, the dissipation ε. Since a dynamic equation for the latter that includes the physical processes relevant to the ocean is still not available, one must resort to different sources of information outside the RSM to obtain a complete *Mixing Scheme* usable in OGCMs.

As for the RSM results, we show that the Γ_α's are functions of both Ri and R_ρ (Richardson number and density ratio representing double diffusion, DD); the Γ_α are different for heat, salt and momentum; in the case of heat, the traditional value $\Gamma_h = 0.2$ is valid only in the presence of strong shear (when DD is inoperative) while when shear subsides, NATRE data show that Γ_h can be three times as large, a result that we reproduce. The salt Γ_s is given in terms of Γ_h. The momentum Γ_m has thus far been guessed with different prescriptions while the RSM provides a well defined expression for $\Gamma_m(Ri,R_\rho)$. Having tested Γ_h, we then test the momentum Γ_m by showing that the turbulent Prandtl number Γ_m/Γ_h vs. Ri reproduces the available data quite well.

As for the dissipation ε, we use different representations, one for the mixed layer (ML), one for the thermocline and one for the ocean's bottom. For the ML, we adopt a procedure analogous to the one successfully used in PB (planetary boundary layer) studies; for the thermocline, we employ an expression for the variable εN^{-2} from studies of the internal gravity waves spectra which includes a latitude dependence; for the ocean bottom, we adopt the enhanced bottom diffusivity expression used by previous authors but with a state of the art internal tidal energy formulation and replace the fixed $\Gamma_\alpha = 0.2$ with the RSM result that brings into the problem the Ri, R_ρ dependence of the Γ_α; the unresolved bottom drag, which has thus far been either ignored or modeled with heuristic relations, is modeled using a formalism we previously developed and tested in PBL studies.

We carried out several tests without an OGCM. Prandtl and flux Richardson numbers vs. Ri. The RSM model reproduces both types of data satisfactorily. DD and Mixing efficiency $\Gamma_h(Ri,R_\rho)$. The RSM model reproduces well the NATRE data. Bimodal ε-distribution. NATRE data show that $\varepsilon(Ri < 1) \approx 10\varepsilon(Ri > 1)$, which our model reproduces. Heat to salt flux ratio. In the $Ri \gg 1$ regime, the RSM predictions reproduce the data satisfactorily. NATRE mass diffusivity. The z-profile of the mass diffusivity reproduces well the measurements at NATRE. The local form of the mixing scheme is algebraic with one cubic equation to solve.

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1. Introduction

In two previous studies (Canuto et al., 2001, 2002, cited as I and II), two vertical mixing schemes for coarse resolution OGCMs (ocean general circulation models) were derived and tested. However, because of shortcomings in I, II of both physical and structural nature, a new mixing scheme became necessary which we present

* Corresponding author at: NASA, Goddard Institute for Space Studies, New York, NY, 10025, USA. Tel.: +1 212 6785571; fax: +1 212 6785560.
E-mail addresses: vcanuto@giss.nasa.gov, vmcanuto@gmail.com (V.M. Canuto).

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here. By structural we mean that the expressions for the heat, salt and momentum diffusivities in I, II were rather cumbersome. By physical, we mean the need to include important physical processes that were missing in I, II.

Concerning the structural issue, we have found a new solution of the *Reynolds Stress Model*, RSM, that yields expressions for the diffusivities that are simpler and thus easier to code than the ones in II. If we denoted by K_α the diffusivities for momentum, heat and salt (subscript α), the new solutions of the RSM are:

$$\text{Mixed layer}: \quad K_\alpha = S_\alpha \frac{2K^2}{\varepsilon}, \tag{1a}$$

$$\text{Deep Ocean}: \quad K_\alpha = \Gamma_\alpha \frac{\varepsilon}{N^2}, \quad \Gamma_\alpha \equiv \frac{1}{2}(\tau N)^2 S_\alpha \tag{1b}$$

Here, K is the eddy kinetic energy, ε its rate of dissipation, N is the Brunt–Vaisala frequency with $N^2 = -g\rho^{-1}\rho_z$, $\tau = 2K\varepsilon^{-1}$ is the dynamical time scale and S_α are dimensionless *structure functions* which are functions of:

$$S_\alpha(Ri, R_\rho, \tau N) \tag{2a}$$

where the Richardson number Ri and the density ratio R_ρ (characterizing double diffusion DD processes) are defined as follows:

$$Ri = \frac{N^2}{\Sigma^2}, \quad R_\rho = \frac{\alpha_s \partial S/\partial z}{\alpha_T \partial T/\partial z} \tag{2b}$$

Here, the variables T, S and $\mathbf{U}$ represent the mean potential temperature, salinity and velocity. The thermal expansion and haline contraction coefficients $\alpha_{T,s} = (-\rho^{-1}\partial\rho/\partial T, +\rho^{-1}\partial\rho/\partial S)$ may be computed using the non-linear UNESCO equation of state and $\Sigma = (2S_{ij}S_{ij})^{1/2}$ is the mean shear with $2S_{ij} = U_{i,j} + U_{j,i}$, where the indices $i, j = 1,2,3$ and $a_{,i} \equiv \partial a/\partial x_i$. Relations (1a) and (1b) contain two unknown variables, the dissipation ε and the eddy kinetic energy K:

$$\varepsilon, \qquad \tau = \frac{2K}{\varepsilon} \tag{3}$$

which means that to complete the RSM, one must add two more relations that provide the variables (3). In engineering flows, these two variables are traditionally obtained by solving the so-called K–ε model which means two differential equations for those two variables. The solution of the K–ε model, represented by Eq. (20), would close the problem since every variable would now be expressed in terms of the large scale fields. Let us analyze how these two variables are determined in the present oceanic context.

1.1. Determination of τ

Since most of the ocean is stably stratified, the vertical extent of the eddies is much smaller than the vertical scale of density variation (except of course in deep convection places), a local approach to the kinetic energy equation, first relation in Eq. (20), is a sensible one. Physically, this is equivalent to taking production equal dissipation, $P = \varepsilon$, where $P = P_s + P_b$ is the total production due to shear and buoyancy. Since $P = K_m\Sigma^2 - K_\rho N^2$, the derivation is presented in Eqs. (22), (23), (54) and (55), use of relations (1b) in $P = \varepsilon$ transforms the latter into an algebraic equation for the variable τ given by Eqs. (40) and (41) the result of which is the function:

$$\tau = \tau(Ri, R_\rho) \tag{4}$$

Use of (4) in the second of (1b) and in (2a) yields the structure functions and the *mixing efficiencies* in terms of the large scale variables:

$$S_\alpha(Ri, R_\rho), \quad \Gamma_\alpha(Ri, R_\rho) \tag{5}$$

Let us note that the above procedure applies in principle to the mixed layer, the thermocline and the ocean bottom. The problem is to know how to determine the Richardson number in each region, a problem we discuss in Sections 6 and 7.3. When applied to the mixed layer, the above determination of the mixing efficiencies is physically equivalent to assuming that the external wind directly generates oceanic mixing. There is, however, a second possibility, namely that the wind first generates surface waves which then become unstable and break, generating mixing (Craig and Banner, 1994; Umlauf and Burchard, 2005). To account for such a process, one needs the full K-equation in (20) with a non-zero flux F_K of K for which one needs a closure. The K-flux F_K is a third-order moment and, as discussed in Cheng et al. (2005), there is still a great deal of uncertainty on how to close such higher-order moments. The wave breaking phenomenon is introduced into the problem by taking the value of F_K at the surface $z = 0$ equal to the power provided by the wave breaking model, as described in the two references just cited. In the present case, local limit $P = \varepsilon$, relations (5) are still not sufficient to determine the diffusivities given by the first relation in (1b) for we require the dissipation ε whose determination we discuss next.

1.2. Determination of ε

In principle, one could solve the second of Eq. (20) and obtain the dissipation $\varepsilon(Ri,R_\rho)$ in analogy with the procedure that lead to relations (5). Regrettably, such a procedure is not feasible since the equation for ε has been problematic since the RSM was first employed by Mellor and Yamada (1982). The reason is that, contrary to the K-equation whose exact form can be derived from turbulence models, the ε-equation has thus far been entirely empirically based and a form that includes stable stratification, unstable stratification and double diffusion, does not exist in the literature. Recently, some progress has been made in deriving an ε-equation from first principles (Canuto et al., 2010) but only for the case of unstable stratification, while most of the ocean is stably stratified. For these reasons, we still cannot employ the dynamic equation for ε and we must rely on a different approach. As for the *mixed layer*, we shall employ the length scheme discussed in Section 6, leading us to relations (62)–(64).[1] In the *thermocline*, we borrow from the IGW (internal gravity waves) studies-parameterizations by several authors (Polzin et al., 1995; Polzin, 1996; Kunze and Sanford, 1996; Gregg et al., 1996; Toole, 1998) the form of ε, more precisely, of εN^{-2}, that contains the dependence on latitude given by Eqs. (65)–(68) which should lead to a sharper tropical thermocline. As for the *ocean bottom*, first we include the enhanced bottom diffusivity due to tides, Eq. (70) as suggested by previous authors but with the latest representation of the function $E(x,y)$ (Jayne, 2009), as well as relation (5) instead of the value $\Gamma = 0.2$ used in all previous studies (St. Laurent et al., 2002; Simmons et al., 2004; Saenko and Merryfield, 2005); second, the tidal drag given by Eq. (72) contains a tidal velocity which thus far has been taken to be a constant while we suggest it should be computed consistently with the same tidal model that provides the function $E(x,y)$, as we explicitly discuss in the lines after Eq. (72); third, the component of the tidal field not aligned with the mean velocity cannot be modeled as a tidal drag. Since its mean shear is large, it gives rise to a *large unresolved shear*with respect to the ocean's bottom. This process, which lowers the local Ri below $Ri = O(1)$ allowing shear instabilities to enhance the diffusivities, was recognized only in one work by Lee et al. (2006) who employed an empirical expression for it. Rather, we adopt the knowledge we acquired in dealing with the same problem in the PBL (Cheng et al., 2002) which gives rise to relation (73) which was tested and assessed in previous work and which was shown to work pretty well.

[1] The 1D-GOTM ocean model (Burchard, 2002) has included and solved the ε-equation in the mixed layer.