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A new approximation for nonlinear wave–wave interactions

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A B S T R A C T

Modern wave models require an accurate computation of the nonlinear wave–wave interactions. This is because nonlinear wave–wave interactions play an important role in the evolution of wind waves, accounting for nonlinear transfer of wave energy to lower and higher frequencies within the spectrum. Presently, in almost all operational state-of-the-art wave models, nonlinear transfer due to wave–wave interactions are evaluated by the discrete interaction approximation (DIA), which was developed by pioneering studies led by Hasselmann more than two decades ago. Although many efforts have tried to develop new methodologies to improve DIA, its basic formulation has not changed. In this study, we present a new computational method by evaluating the dominant nonlinear wave transfer along the wavenumber and the wave directional axes, and by approximating the contributions along the resonance loci. The new method is denoted the Advanced Dominant Interaction (AvDI) method. We show that AvDI is sufficiently efficient that it can be implemented within an operational wave model. As a validation of the approach, we compare simulations of hurricane Juan with observed wave data.

1. Introduction

In recent years, the simulation and forecasting of intense cyclones and their associated maximum waves have become important issues in coastal ocean waters, due to the increased population living in these areas and the increase in potential damage to human development and societal infrastructure. Large, complex ocean waves can be generated by marine storms and their rapidly-varying winds and they can propagate thousands of kilometers from their generation centers to coastal areas. An accurate efficient computation of nonlinear wave–wave interactions is an important key to getting reliable wave forecasts.

While numerical modeling has made impressive steps in forecasting waves on global and regional scales and considerable efforts have been made to accurately simulate and measure directional wave spectra generated by marine storms, progress in the development of operational algorithms for evaluating the nonlinear wave–wave interactions has not been as rapid. Almost all modern operational wave models implemented on large-scale lakes and oceans use the discrete interaction approximation (DIA) formulation given by Hasselmann and Hasselmann (1985) and WAMDI (1988).

This paper presents a new method to compute the nonlinear wave–wave interactions. The new method is based on the Webb–Resio–Tracy algorithm (hereafter WRT), which has been described by Webb (1978), Tracy and Resio (1982), Resio and Perrie (1991, 2008) and Van Vledder (2006). The WRT method uses scaling similarities to reduce the number of computations and thereby speed up the overall computation. We suggest that this new method is a potential candidate for further development and application in operational wave forecast models.

We start with the well-known action $N(f,\theta)$ balance equation for wind-generated waves (Komen et al., 1994). In terms of wavenumber and direction, the action density may be written as $N(k,\theta,\phi,\lambda)$ and the conservation equation is generally expressed as,

$$\frac{\partial N}{\partial t}+\frac{1}{\cos\ \phi}\frac{\partial}{\partial\phi}\dot{\phi}N\ \cos\ \theta+\frac{\partial}{\partial\lambda}\dot{\lambda}N+\frac{\partial}{\partial k}\dot{k}N+\frac{\partial}{\partial\theta}\theta_g N=\frac{S}{\sigma} \quad (1)$$

where

$$\dot{\phi}=\frac{c_g\ \cos\ \theta+U_\phi}{R}$$
$$\dot{\lambda}=\frac{c_g\ \sin\ \theta+U_\lambda}{R\ \cos\ \phi}$$
$$\dot{k}=-\frac{\partial\sigma\partial d}{\partial d\partial s}-\mathbf{k}\cdot\frac{\partial\mathbf{U}}{\partial s}$$
$$\dot{\theta}_g=\dot{\theta}-\frac{c_g\ \tan\ \phi\ \cos\ \theta}{R}$$
$$\dot{\theta}=-\frac{1}{k}\left(\frac{\partial\sigma\partial d}{\partial d\partial m}-\mathbf{k}\cdot\frac{\partial\mathbf{U}}{\partial m}\right) \quad (2)$$

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where ϕ is latitude, λ is longitude, θ is the direction of wave propagation, s is a coordinate parallel to θ and m is a coordinate perpendicular to θ, σ is the angular frequency, R is the radius of the earth, and $U_{\phi,\lambda}$ is the ocean current component in ϕ and λ directions, respectively.

On the right side of Eq. (1), S is the net source term consisting of wind input (S_{in}), nonlinear quadruplet wave–wave interactions (S_{nl}), wave-breaking dissipation (S_{ds}) and bottom friction (S_{bot}). The nonlinear interactions (S_{nl}) are important because they distribute spectral energy to higher and lower frequencies, and directionally within the spectrum. In this paper, we focus on the nonlinear wave–wave interactions (S_{nl}), which are conservative, neither creating nor dissipating energy.

In a pioneering study, Hasselmann (1962) derived an analytic expression for S_{nl}, which is often referred to as the Boltzmann integral or kinetic equation. Some time later, Hasselmann and Hasselmann (1981) presented the Exact-NL formulation to numerically estimate S_{nl}. This method was the first systematic algorithm for this problem. However, this approach is too time-consuming for operational wave forecasting. Therefore, several years later Hasselmann et al. (1985) developed the Discrete Interaction Approximation (DIA), with dramatically increased computational efficiency compared to Exact-NL. The development of DIA allowed the formulation of third-generation wave prediction models, such as WAM, WAVEWATCHIII and SWAN. However, DIA has a number of well-known shortcomings and for many types of spectra compares poorly with a full evaluation of S_{nl} (Van Vledder, 2001; Resio and Perrie, 2008; Perrie and Resio, 2009).

In recent years, several attempts have been made to formulate a more efficient, accurate parameterization for S_{nl} by incrementally simplifying the "exact" WRT method. Lin and Perrie (1999) suggested a reduced integration approach. Several studies have tried to move beyond the basic DIA approach, expanding DIA, or using multiple representative quadruplets (Krasnopolsky et al., 2002; Tolman and Krasnopolsky, 2004; Tolman et al., 2005; Van Vledder, 2001, 2006; Tolman, 2004; Hashimoto and Kawaguchi, 2001). Recently, a two-scale approximation to wave–wave interactions has been suggested by Resio and Perrie (2008) and Perrie and Resio (2009).

Motivated by Tracy and Resio (1982), Susilo and Perrie (2007) developed an algorithm that estimates a scaling factor to evaluate the nonlinear transfer, based on the largest contributions, or dominant contributions, to S_{nl} along the mean wave direction. This method achieves a reduction in computational time by selecting sets of interacting wavenumbers that produce the dominant transfers so that it is not necessary to compute the integral for the entire spectrum. However, the method needs additional optimization before it can be applied for operational forecast models.

In this study, a modern operational third-generation spectral wave model is used to test the new AvDI formulation for nonlinear wave–wave interactions, WAVEWATCH III (hereafter WW3) version 1.18 (Tolman, 1999, 2002). WW3 includes numerical and physical parameterizations that make it suitable for a large range of scales including global, ocean-basin scale, shelf scales, and high-resolution coastal ocean regions. We first present a theoretical development of the AvDI method in Section 2, based on the WRT methodology. As a practical demonstration, AvDI is implemented in WW3 in Section 3. Results from numerical experiments, involving both JONSWAP wave observations and a real storm case are described in Section 4. Tests involving storm-generated waves are important because parameterizations for S_{nl} have sometimes been found to perform much better for JONSWAP spectra than for evolutionary storm cases (Tolman, 2004). In this study, the storm is hurricane Juan which made landfall in Halifax, Nova Scotia on September 29, 2003 as a category two hurricane. Model validation is based on wave buoy observations. Conclusions are given in Section 5.

2. Theoretical and numerical development

The basic equation describing the nonlinear quadruplet wave–wave interactions (Hasselmann, 1962; Zakharov and Filonenko, 1966) is known as the full Boltzmann integral (FBI). This relation gives the rate of change of action density S_{nl}, due to all resonant interactions among quadruplets of wave numbers. It may be expressed as

$$\frac{dN_1}{dt} = \int\int\int C^2(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)D(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) \times\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4)\delta(\omega_1+\omega_2-\omega_3-\omega_4)d\mathbf{k}_2d\mathbf{k}_3d\mathbf{k}_4 \quad (3)$$

where

$$D(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = N_1N_3(N_4-N_2)+N_2N_4(N_3-N_1) \quad (4)$$

and where N_1 is the action density at wave number $\mathbf{k}_1$. Webb (1978) expressed this equation in terms of a transfer function $T(\mathbf{k}_1,\mathbf{k}_3)$ where

$$\frac{dN_1}{dt} = 2\int T(\mathbf{k}_1,\mathbf{k}_3)d\mathbf{k}_3 = 2\int_0^\infty\int_0^{2\pi} T(\mathbf{k}_1,\mathbf{k}_3)\ \mathbf{k}_3d\theta_3d\mathbf{k}_3 \quad (5)$$

and

$$T(\mathbf{k}_1,\mathbf{k}_3) = \int\int C^2(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)D(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) \times\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4)\delta(\omega_1+\omega_2-\omega_3-\omega_4) \times\Theta(|\mathbf{k}_1-\mathbf{k}_4|-|\mathbf{k}_1-\mathbf{k}_3|)d\mathbf{k}_2d\mathbf{k}_4. \quad (6)$$

Here, ω_i is the angular frequency at $\mathbf{k}_i$, $\delta(\ldots)$ is the Dirac delta function, C^2 is the coupling coefficient (Webb, 1978; Tracy and Resio, 1982) and Θ is the Heaviside function,

$$\Theta(x) = 1 \quad \text{if} \quad x > 0$$
$$\Theta(x) = 1 \quad \text{if} \quad x \leqslant 0$$
$$x = |\mathbf{k}_1-\mathbf{k}_4|-|\mathbf{k}_1-\mathbf{k}_3|. \quad (7)$$

Applying the resonance conditions $\omega_1+\omega_2=\omega_3+\omega_4$ and $\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}_3+\mathbf{k}_4$, Tracy and Resio (1982) and Resio and Perrie (1991) restated the transfer integral (Eq. (6)) as

$$T(\mathbf{k}_1,\mathbf{k}_3) = \int_C C^2D\left|\frac{\partial W}{\partial\mathbf{n}}\right|^{-1}\Theta(\mathbf{k}_1,\mathbf{k}_3,\mathbf{k}_4)d\mathbf{s} \quad (8)$$

which is a contour integral. Here, $W = \omega_1+\omega_2+\omega_3+\omega_4$, the frequency resonance condition is $W = 0$, unit vector $\mathbf{s}$ is along the interaction locus, and unit vector $\mathbf{n}$ is normal to that locus. In terms of a density function $D(N)$ and a geometry function $G(\mathbf{k})$, Eq. (8) may be written as

$$T(\mathbf{k}_1,\mathbf{k}_3) = \int_C D(N)G(\mathbf{k}) \quad (9)$$

where

$$G(k) = C^2\left|\frac{\partial W}{\partial n}\right|^{-1}\Theta(\mathbf{k}_1,\mathbf{k}_3,\mathbf{k}_4)d\mathbf{s}. \quad (10)$$

In evaluating the full Boltzmann integral, Eq. (5) may be expressed as

$$\frac{dN_1}{dt} = \int_0^\infty\int_0^{2\pi}\int_C \ldots\ ds d\theta_3 d\mathbf{k}_3 \quad (11)$$

where it is important to include all contributions from the entire domain of the wave spectra including all possible resonance combinations satisfying the interaction loci. If there are i frequency bins, j angle bins and l loci bins, the integral requires $i\times j\times l$ calculations to compute dN_1/dt, compared to DIA which requires $i\times j$