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A simple mixing scheme for models that resolve breaking internal waves

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ABSTRACT

Breaking internal waves in the vicinity of topography can reach heights of over 100 m and are thought to enhance basin-wide energy dissipation and mixing in the ocean. The scales at which these waves are modelled often include the breaking of large waves (10 s of meters), but not the turbulence dissipation scales (centimeters). Previous approaches to parameterize the turbulence have been to use a universally large viscosity, or to use mixing schemes that rely on Richardson-number criteria.

A simple alternative is presented that enhances mixing and viscosity in the presence of breaking waves by assuming that dissipation is governed by the equivalence of the density overturning scales to the Ozmidov scale ($\varepsilon = L_T^2 N^3$, where L_T is the size of the density overturns, and N the stratification). Eddy diffusivities and viscosities are related to the dissipation by the Osborn relation ($K_z = \Gamma \varepsilon N^{-2}$) to yield a simple parameterization $K_z = \Gamma L_T^2 N$, where $\Gamma \approx 0.2$ is the flux coefficient. This method is compared to previous schemes for flow over topography to show that, when eddy diffusivity and viscosity are assumed to be proportional, it dissipates the correct amount of energy, and that the dissipation reported by the mixing scheme is consistent with energy losses in the model. A significant advantage of this scheme is that it has no tunable parameters, apart from the turbulent Prandtl number and flux coefficient. A disadvantage is that the overturning scales of the turbulence must be relatively well-resolved.

1. Introduction

There is considerable interest in how topography interacts with stratified flows to produce internal waves and turbulence. In particular the role of internal waves produced by tides and lee waves in flows over topography have been examined. In general these have been treated with large-scale general circulation models (i.e., POM Merrifield and Holloway (2002) or ROMS), or with specialized non-hydrostatic codes (MITgcm Legg and Adcroft (2003), SUNTANS Venayagamoorthy and Fringer (2005)). At the other end of the spectrum LES or DNS simulations have been carried out on small scales that resolve turbulence.

Here we are interested in the scales typical of flow over deep ocean ridges, like Hawaii (Klymak et al., 2008), and at continental slopes, like Oregon (Nash et al., 2007). These flows are deep, up to 4000 m, and exhibit features on a variety of scales, from low-mode internal tides that span the whole water column, to breaking nonlinear waves near abrupt changes in the topographic slopes. These breaking waves lead to density inversions that can be over 150-m tall, with dissipation rates $\varepsilon > 5 \times 10^{-8}$ m^2 s^{-3} in stratifications $N^2 \approx 10^{-6}$ s^{-2}. These flows have turbulent Reynolds numbers exceeding 10^6, and buoyancy Reynolds numbers $Re_b = \varepsilon/\nu N^2 > 10^4$, and Kolmogorov scales on the order of 10^{-3} m. To capture the full range of scales would require 10^6 grid cells in each dimension. Instead, to study these phenomena, we have made extensive use of a relatively efficient class of 2-D simulations that allow good resolution in the vertical (O(10 m)) and horizontal (O(100 m)), such that the large-scale forcing and subsequent breaking of internal waves can be simulated. Direct numerical simulation methods are prohibitively expensive for exploration of the parameter spaces in which these phenomena are forced, therefore the turbulence dissipation in these simulations needs to be parameterized.

Two approaches to parameterizing turbulence at this scale of modelling have been used. The first is to use a high vertical viscosity $A_z \sim 10^{-2}$ m^2 s^{-1} (Legg and Klymak, 2008) or $A_z \sim 10^{-1}$ m^2 s^{-1} (Legg and Huijts, 2006; Legg and Adcroft, 2003). This has the benefit of being simple, and yielding the turbulence dissipation in the flow

$$\varepsilon = A_z \left(\frac{\partial \mathbf{u}}{\partial z}\right)^2 + A_h \left(\frac{\partial \mathbf{u}}{\partial x}\right)^2 + A_h \left(\frac{\partial \mathbf{u}}{\partial y}\right)^2. \quad (1)$$

As we argue below, this scheme depends strongly on the choice of A_z. Even if it is tuned to the breaking waves, it can exaggerate dissipation in the midfield where the shear is strong but not strong enough to excite shear instability or breaking.

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The alternative has been schemes that have enhanced mixing based on the value of the gradient Richardson number $Ri = (\partial U_h/\partial z)^{-2}N^2$ (where U_h is the horizontal component of velocity, and N the buoyancy frequency). Some schemes depend on a critical Richardson number below which the turbulence is increased, such as the Mellor–Yamada scheme used here (Mellor and Yamada, 1982), while more recent schemes remove the necessity for a critical Richardson number (Galperin et al., 2007; Canuto et al., 2008). In all such schemes the production rate of turbulence kinetic energy is assumed to be $P = A_z(\partial U_h/\partial z)^2$ where A_z is a turbulent vertical diffusivity meant to represent unresolved eddies. The problem with these schemes in simulations with resolved breaking waves is that the turbulent eddies are partially resolved and drive overturning so that $Ri^{-1} < 0$ is resolved by the model. All the schemes introduce an arbitrarily large viscosity for negative Richardson number and ε calculated from the local shear can be unreasonably high or low, depending on this arbitrary choice.

Here we present a simple local scheme for mixing in breaking regions based on the observed correlation between the size of the convecting overturn and the Ozmidov scale. The Ozmidov scale is related to the rate of turbulence dissipation by $\varepsilon = L_o^2 N^{-3}$. Energy arguments and observational evidence indicates that the size of convectively unstable vertical displacements in a turbulent patch L_T, is approximately equivalent to the Ozmidov scale: $L_t \approx L_o$ (Dillon, 1982; Wesson and Gregg, 1994; Moum, 1996). This brushes over significant changes in the dissipation during the life of an overturn (i.e., Gargett and Moum, 1995; Smyth et al., 2001), but is a rough average dissipation rate. The correspondence between dissipation rates and the size of overturns in convective instabilities like those found over Hawaii or in fjords appears to be robust (Klymak and Gregg, 2004; Klymak et al., 2008). We present this scheme as a bridge between large scale models that do not resolve breaking waves, and small-scale large-eddy or direct numerical simulations.

Below we implement this simple scheme whereby the turbulent viscosity A_z is calculated from the size of overturns driven by the breaking waves. We compare the energy dissipation predicted by the parameterized A_z to the energy lost from the model for the proposed scheme and compare to the constant-A_z runs and those using the Mellor–Yamada parameterization, a widely used Richardson-number scheme. Two idealized cases of interest are considered, steady and oscillating tidal flow over an obstacle.

2. Methods

The model used here is the MITgcm (Marshall et al., 1997; Legg and Klymak, 2008). We use a 2-dimensional topography, with a stretched horizontal co-ordinate system. For most of the runs here water depth $H = 2000$ m, and vertical resolution was 200 points with $\delta z_m = 10$ m; a few runs were made with $H = 1300$ and 1650 m. The horizontal domain was 174 km over 240 grid cells. The inner 80 grid cells were spaced 100 m apart, and then the grid was telescoped linearly so that for the outer cells $\Delta x = 2$ km. The obstacle in all cases is a Gaussian shape, height from the seafloor given by $h = h_m \exp(-x^2/W^2)$. The width W introduces an aspect ratio to the problem $\alpha_o = h_m/W$. The model was run in hydrostatic mode for numerical efficiency. Experiments with non-hydrostatic code did not reveal substantive differences in the features of interest here (see Section 3.2 below for a comparison). That is somewhat counterintuitive, since our proposed scheme depends on breaking waves to provide the turbulence. However, the non-hydrostatic terms in the vertical momentum equation only contribute to the evolution of the breaking, not to its actual onset. This would require a considerably more isotropic simulation grid than desirable for these scales, and would be amenable to a more isotropic mixing scheme as well. For most runs, horizontal viscosities and diffusivities were kept as low as numerically feasible, at 10^{-4} m^2 s^{-1}, except where noted.

2.1. Vertical turbulence schemes

2.1.1. Constant viscosity

These runs compare with Legg and Huijts (2006) and Legg and Klymak (2008) who used high vertical and horizontal viscosities $A_z = 10^{-2}$ m^2 s^{-1}, and $A_H = 10^{-1}$ m^2 s^{-1}. In those papers, explicit mixing was set to zero, and handled numerically by a Superbee advection scheme (van Leer, 1979). In all our simulations shown here, the diffusivity was set to be the same as the viscosity, except for a sensitivity study that shows small differences in the dissipation due to the higher-order scheme (Section 4). The constant viscosity runs have the simple advantage that dissipation is relatively easy to compute from the flow field and local shears, and so long as the model is well-resolved, gives an accurate representation of the simulated dissipation. The disadvantage to this scheme is that the Reynolds number can be too low to allow turbulent features to develop in the first place, and that it can place too much dissipation in regions that are not expected to be turbulent.

2.1.2. Mellor–Yamada, 2.0

The Mellor–Yamada formulation used by the MITgcm is a second-order local model. This version of the scheme does not solve a prognostic equation for the turbulent kinetic energy (so this is not to be confused with what is commonly called "Mellor–Yamada 2.5"). This scheme enhances viscosity above an arbitrary background of $A_z = 10^{-5}$ m^2 s^{-1} if $Ri = N^2/S^2 < 0.25$, and viscosity becomes very high if $Ri < 0$ in density overturns (Fig. 1). With no capping, the maximum viscosity is 4×10^{-1} m^2 s^{-1}. The implementation used here allows a cap to this value, adding an adjustable parameter A_{max}. The production of turbulent kinetic energy implied by this relationship between viscosity and Richardson number is the same as for the more elaborate MY2.5, but in the local scheme energy is assumed to be dissipated locally rather than propagating to remote grid cells; in this paper we refer to this as MY2.0. While we used MY2.0 rather than MY2.5 because that was the scheme already implemented in the MITgcm, it is likely that similar results would be found with MY2.5 because the production of turbulence is the same in both schemes, and this production term, rather than the diffusion and advection of TKE, is the principal difference introduced by our new scheme below. Similarly the choice of critical Richardson number has little influence on our comparison, since the flaws in the MY schemes which we are addressing here occur for $Ri < 0$.

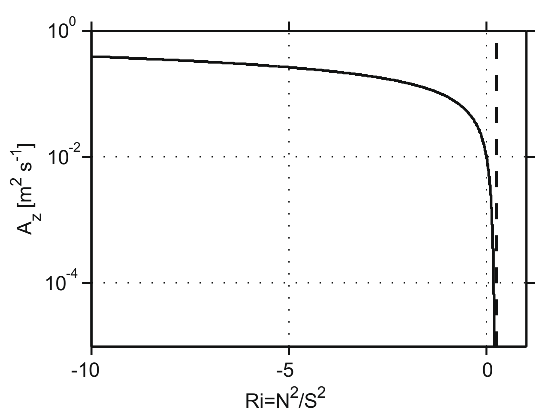

Fig. 1. Mellor–Yamada 2.0 scheme used in the MITgcm. Negative Richardson numbers mean that the stratification is unstable. The dashed line is $Ri = 0.25$. The background value of $A_z = 10^{-5}$ m^2 s^{-1}.