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# Alternating zonal jets and energy fluxes in barotropic wind-driven gyres

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ABSTRACT

The barotropic $\beta$-plane vorticity equation is considered under steady large scale (double-gyre) and small scale (stochastic) forcing. For both forcings, regimes are found in which alternating zonal jets appear. For steady large scale forcing, this regime is characterized by weak forcing and weak dissipation. Attention is focused on energy cascades due to the nonlinear and $\beta$ terms and the jets are found to be associated with to a near compensation in these cascades over a range of wavenumbers. Additionally, interaction between flow forced at large scale and flow forced at small scale is examined.

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## 1. Introduction

Recent evidence has unmasked the presence of alternating zonal jet-like features superimposed on the larger scale mid-latitude oceanic gyre circulation (Treguier et al., 2003; Nakano and Hasumi, 2005; Maximenko et al., 2005; Richards et al., 2006; Maximenko et al., 2008). While the exact mechanism underlying the formation of these features is still not well understood, a number of possible mechanisms have been suggested. For example, alternating jets are well-known from $\beta$-plane turbulence and are associated with a halting of the two-dimensional inverse energy cascade by Rossby wave dispersion (e.g., Rhines, 1975; Vallis and Maltrud, 1993; Panetta, 1993) . Thus, it is possible that the recently observed features are consistent with the classic picture of ocean energetics whereby barotropization of mesoscale eddies acts as an effective small scale energy source for the barotropic flow (e.g., Salmon, 1998; Vallis, 2006). A subsequent inverse energy cascade to the Rhines scale would then produce the jets.

However, the reflection of long Rossby waves as short Rossby waves at a western boundary and the attendant inviscid generation of anisotropy can lead to dynamics that are different from those in the zonally periodic setting. For example, in the case with meridional boundaries, a linear forward cascade of energy, associated with the $\beta$ term, is possible, whereas that is not the case in a periodic setting. Nevertheless, idealized numerical studies (Kramer et al., 2006; Nadiga, 2006) in a closed basin suggest that the anisotropization-of-inverse-cascade mechanism can survive in the presence of meridional boundaries, as is relevant for the oceanic context. Other suggestions for the formation of oceanic jets include nonlinear self-interactions of linear eigenmodes (Berloff, 2005; Berloff et al., 2009) and that such jets are preferred growing structures excited by the imposed stochastic forcing (Farrell and Ioannou, 2007). Finally, zonal jets can be formed simply by the instability of barotropic Rossby waves (Lorenz, 1972; Gill, 1974; Connaughton et al., in press).

Both the $\beta$-plane turbulence and the gyre scale dynamics are nonlinear and it seems reasonable to anticipate that the two will interact. A full description of the interactions will clearly necessitate consideration of baroclinic effects; it nonetheless seems useful to begin by understanding how jets interact with the wind-driven circulation in the simpler context of the purely barotropic problem. In this paper, we consider the $\beta$-plane barotropic vorticity equation in a box geometry forced by (i) a steady large scale wind, (ii) a small scale stochastic forcing and (iii) both. The first case is the classic mid-latitude double gyre problem. The second has previously been used to model the jets (Nadiga, 2006; Kramer et al., 2006). It might be thought of as a crude model of energy being injected by small scale baroclinic eddies into the barotropic mode. The third allows us to consider interactions between the two. We focus primarily on a description of the energy cascades.

In Section 2, we describe the model and introduce diagnostics. Section 3 gives results for the three types of forcing. We first describe a regime in which jets appear in the barotropic double gyre problem. The jets are shown to be associated with a near compensation between a (linear) forward energy cascade related to the $\beta$ effect in the presence of meridional boundaries and the nonlinear

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inverse cascade familiar from two-dimensional turbulence. Next, consistent with previous studies, we also find jets to appear in response to small scale stochastic forcing. Note that in both cases, the jets are visible only after averaging. In instantaneous snapshots, they are masked by the presence of a stronger isotropic eddy field. Finally, interaction between flow forced at small and large scale is considered. A discussion in Section 4 is then followed a brief summary in Section 5.

## 2. Model, diagnostics and background

### 2.1. Equations, parameters and diagnostics

We solve the barotropic quasigeostrophic potential vorticity equation:

$$\frac{\partial q}{\partial t} = -J(\psi, q) + F - r\nabla^2\psi + A\nabla^8\psi \qquad (2.1)$$
$$q = \nabla^2\psi + \beta y$$

The model domain is taken to be a square box of width $2\pi L$. Forcing, $F$, includes a large scale wind forcing $F_{\text{large}} = F_0 \sin(y/L)$ and a small scale stochastic forcing, $F_{\text{small}}$. $F_{\text{small}}$ injects energy at a average rate, $\epsilon$, into a narrow ring of wavenumbers centered on a large wavenumber, $k_f$. This forcing is stochastic in time spatially homogeneous and present in a narrow ring of wavenumbers centered around a $k_f$ of 128. Time dependence in $F_{\text{small}}$ is given by an Ornstein–Uhlenbeck process. The decorrelation time corresponds to about 4% of an estimate of the turnover time, $L/U_{rms}$. A consequence of this choice is that $\epsilon$ is not known *a priori*, but must be measured. We have conducted a few of the stochastically forced simulations with white noise forcing instead of the colored forcing and verified that our results remain the same. At lateral boundaries, a generalization of slip conditions appropriate for hyperviscosity is applied. Specifically, we set $\nabla^2\psi = \nabla^4\psi = \nabla^6\psi = 0$. With this choice, boundary terms that might otherwise appear in the energy equation are eliminated.

An equation for the domain-averaged energy, $E$, is found by multiplying (2.1) by $\psi$ and averaging over the domain:

$$\frac{dE}{dt} = \text{Forcing} - 2rE - \frac{A}{4\pi^2L^2}\int\int(\nabla^4\psi)^2 dx\,dy. \qquad (2.2)$$

Here, the forcing term is the sum of the energy injection rate due to the small scale stochastic forcing and $W$, the wind power source. $W$ is given by

$$W \doteq \frac{-1}{4\pi^2L^2}\int\int(\psi F_{\text{large}})dx\,dy. \qquad (2.3)$$

In the absence of a forward energy cascade to dissipation wavenumbers, energy dissipation by the hyperviscosity is negligible and statistical equilibrium implies

$$2r\overline{E} = rU_{rms}^2 \approx \epsilon + \overline{W}, \qquad (2.4)$$

where overbars denote time averages.

As mentioned, $\epsilon$ must be determined from the solution since the stochastic forcing used had a finite decorrelation time. Similarly, $\overline{W}$ is also a function of the solution; however, one can make a useful *a priori* estimate by assuming the large scale flow to be well approximated by the Sverdrup balance:

$$\beta\frac{\partial\psi_{\text{Sv}}}{\partial x} = F_{\text{large}}. \qquad (2.5)$$

This gives

$$\overline{W} \approx \frac{F_0^2\pi L}{2\beta}, \qquad (2.6)$$

which is a good approximation, except where damping is weak, in which case a four gyre response tends to develop and $\overline{W}$ decreases (Greatbatch and Nadiga, 2000). Note that (2.4) and (2.6) yield an estimate for the root mean square (rms) velocity at statistical equilibrium.

In general, the problem is defined by the dimensional parameters $\beta, L, F_0, \epsilon, k_f, r$ and $A$. The precise value of $A$ is largely irrelevant. The main role of hyperviscosity is to damp enstrophy variance, which cascades forward to high wavenumbers and the value of $A$ determines the position of a viscous tail in the spectra. Although bottleneck effects can influence the spectra at wavenumbers slightly to the left of the viscous tail (e.g., see Frisch et al., 2008), for sufficiently high resolution (and low $A$), the spectra are insensitive to $A$ over low-to-moderate wavenumbers. We choose $k_f = 128$, so that $k_f^{-1}$ is small compared to 1 and large compared to the viscous scale. To limit the size of our parameter space, $k_f$, $L$, $A$ and $\beta$ will be held fixed.

A range of values for the remaining parameters, $r$, $F_0$ and $\epsilon$ will be considered. It is convenient to express these non-dimensionally. Consider first the case with large scale forcing only. It is common in oceanography to express $r$ and $F_0$ non-dimensionally as (e.g., Pedlosky, 1996):

$$\delta_s \equiv \frac{r}{\beta L}, \quad \delta_i \equiv \frac{F_0^{1/2}}{\beta L}. \qquad (2.7)$$

Physically, $\delta_s$ and $\delta_i$ give the ratio of the linear Stommel layer thickness and an *a priori* estimate of the inertial boundary layer thickness to $L$.[1] Note that $L\delta_i$ can be thought of as a Rhines scale,

$$\delta_i = \frac{1}{L}\left(\frac{U_{\text{Sv}}}{\beta}\right)^{1/2}, \qquad (2.8)$$

where $U_{\text{Sv}} = F_0/\beta$ gives the Sverdrup velocity amplitude. Alternatively, one might wish to consider a Rhines scale based on an estimate of the rms velocity. Taking $\epsilon = 0$ and using (2.4) and (2.6), we define

$$\delta_I \doteq \delta_i\left(\frac{\pi}{2\delta_s}\right)^{1/4} \approx \left(\frac{U_{\text{rms}}}{\beta L^2}\right)^{1/2}. \qquad (2.9)$$

Loosely speaking, $\delta_I$ might be thought of as giving an estimate of the energy-containing eddy scales in the gyre problem. In eddying flows, typically, $\delta_I$ is larger than $\delta_i$.

Given the form of (2.9), it is obvious that the same $\delta_I$ can be achieved by different combinations of forcing $\delta_i$ and dissipation $\delta_s$. Moreover, one might anticipate a difference in behavior between, for example, a case where $\delta_I$ is made large by choosing $\delta_i$ large and one where the same $\delta_I$ is achieved by choosing $\delta_s$ small. One of our principal findings is that, in the latter case (dissipation and forcing both small), zonal jets appear super-posed on the double gyre circulation. To our knowledge, this is the first time jets have been produced in a gyre simulation with neither baroclinity nor small scale forcing.

By analogy with $\delta_I$, we can also define a similar parameter for the small scale forcing:

$$\delta_\epsilon \doteq \frac{1}{L}\left(\frac{\epsilon}{\beta^2 r}\right)^{1/4} \approx \left(\frac{U_{\text{rms}}}{\beta L^2}\right)^{1/2}, \qquad (2.10)$$

where in this case, $U_{\text{rms}} \approx (\epsilon/r)^{1/2}$. For a simulation forced by $F_{\text{small}}$, one expects an inverse cascade feeding energy into zonal jets with a characteristic width that scales like $\delta_\epsilon$. This assumes, of course, that the Rayleigh damping is sufficiently weak so as to allow the inverse energy cascade to reach the Rhines scale, which will typically be the case if $\delta_s$ is small compared to $\delta_\epsilon$.

[1] Note that Pedlosky's definition differs from ours by a factor of $2\pi$.