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Topocaustics

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ABSTRACT

Deep (∼2000 m) observations near the Sigsbee escarpment in the Gulf of Mexico show short-period (approximately 5–12 days) energetic currents due to topographic Rossby waves (TRW's). We suggest that the phenomenon is due to the focusing and accumulation of TRW energy by the slopes coupled with a bend in isobaths, in a topographic caustic (topocaustic). The idea draws on a simple mathematical equivalence between the propagation of internal waves and of TRW's. Topocaustics occur near regions of maximum $N_T = N|\nabla h|$ (N = Brunt–Väisälä frequency; h = water depth). Because of the one-sided propagation property of TRW's, energy also tends to accumulate at the "western" end of closed contours of N_T. The process is demonstrated here using a nonlinear primitive-equation numerical model with idealized bathymetry and forcing. A Gulf of Mexico simulation initialized with a data-assimilated analysis covering the period of the Sigsbee observation is then conducted. The mooring is near a localized maximum N_T, and Intrinsic Mode Functions confirm the existence of energetic bursts of short-period deep-current events. The strong currents are locally forced from above, either by an extended Loop Current or a warm ring.

1. Introduction

Near the Sigsbee escarpment in the Gulf of Mexico (Fig. 1; water depths ≈2000 m), Hamilton and Lugo-Fernandez (2001), Lai and Huang (2005), and Hamilton (2007) observed short-period (approximately 5–12 days) bottom-trapped topographic Rossby waves (TRW's) consisting of bursts of current fluctuations with amplitudes which are approximately 0.2–0.3 m s^{-1} near the bottom (Fig. 1). These intense TRW's appear to emanate from the south and southeast, from frontal eddies of the Loop Current and propagating rings, and also from deep eddies (Oey and Lee, 2002; Hamilton, 2007; Oey, 2008). Across the escarpment, a few kilometers to the northwest (of the mooring shown in Fig. 1), current fluctuations rapidly decay, which may indicate that the TRW's are reflected off the escarpment (Hamilton, 2007). In this work, we use the mathematical equivalence between the propagation of TRW's and internal waves (IW's) to explain the build-up and rapid decay of TRW's near the escarpment.

Topographic Rossby wave energy propagates along rays the direction and (group) speed of which depend on the horizontal wavelength ($2\pi/K$) and period ($2\pi/\sigma$), as well as on environmental parameters such as the water depth (h), depth gradients (∇h), Brunt–Väisälä frequency (N) and also the background mean current ($\mathbf{V}$) and its shears if these exist (Rhines, 1970; Oey and Lee, 2002). A tacit assumption is that these parameters are slowly varying over the TRW wavelengths and periods. Formulae (2) and (3) (Oey and Lee, 2002) below are approximately valid provided that

$$NhK/|f| = h/h_{trap} \approx O(1) \text{ or larger}, \quad (1)$$

where $h_{trap} = |f|/(NK)$ is the trapping depth of TRW's ($h_{trap} \approx 1000$ m for $2\pi/K \approx 100$ km, $f \approx 6.7 \times 10^{-5}$ s^{-1} and $N \approx 10^{-3}$ s^{-1}). Thus,

$$\sigma \approx N_T \sin(\theta), \quad (2)$$

where $N_T = N|\nabla h|$, θ is the clockwise angle the wavenumber vector $\mathbf{K}$ makes with ∇h (see Fig. 1 of Oey and Lee, 2002); also,

$$\mathbf{C}_g \times \mathbf{K} \approx N_T \cos(\theta)\mathbf{n}_u, \quad (3)$$

where $\mathbf{C}_g = \nabla_{\mathbf{K}}\sigma$ is the group velocity, and $\mathbf{n}_u$ is an 'upward' unit vector perpendicular to both x and y. Eq. (2) says that σ depends only on the wavenumber direction θ, but not its magnitude $|\mathbf{K}|$. Therefore $\mathbf{K}.\nabla_{\mathbf{K}}\sigma = 0$ since it is (proportional to) the component in the fixed $\mathbf{K}$-direction of the rate of change of σ in the $\mathbf{K}$-space, and is non-zero only if σ changes with $|\mathbf{K}|$ (Lighthill, 1978). Thus $\mathbf{C}_g$ and $\mathbf{K}$ are perpendicular to each other, and Eq. (3) says moreover that $\mathbf{C}_g$ points upslope when $\mathbf{K}$ points down-slope and vice versa. These properties make TRW's different from planetary Rossby waves, continental shelf waves or coastally trapped waves (Gill, 1982) but more similar to IW's for which the frequency is also a function of the wavenumber direction only. Indeed, TRW-propagation ($f > 0$) is identical to two-dimensional IW-propagation in the left half of the xz-plane, in which the z-direction may be identified as the y-direction pointing in the $\mathbf{n}_{hy}$ ($= -\nabla h/|\nabla h|$) direction (towards deceasing water depth). For IW's the maximum possible frequency of (local) oscillations is N;

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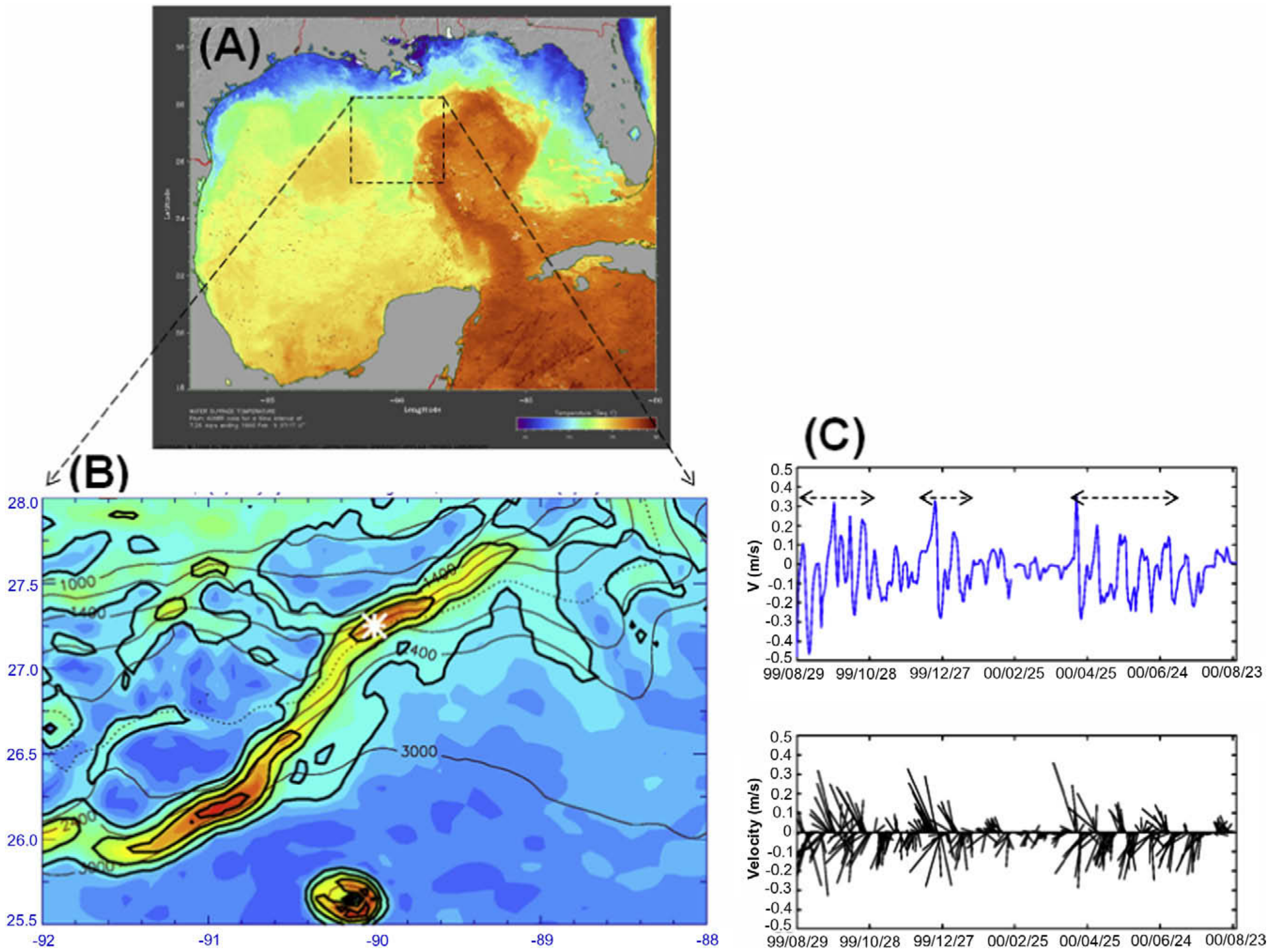

Fig. 1. (A) AVHRR (Advanced Very High Resolution Radiometry; http://fermi.jhuapl.edu/avhrr/gm/averages/) 7-day composite sea-surface temperature in the Gulf of Mexico in February/1993 showing the Loop Current and a warm-core ring further west; (B) contours (dark lines and color shading) of maximum allowable TRW frequency $N_T = N|\nabla h|$ (cycles/day or cpd) in the vicinity of the Sigsbee escarpment (box in panel A). Here the $N = 6 \times 10^{-4}\ \mathrm{s}^{-1}$. Thin brown contours are isobaths, and the dotted line = 2000 m. The escarpment is identified with the band of high N_T oriented northeast to southwest, approximately along the 2000 m isobaths in the north and along the 3000 m isobaths in the south. The "*" is one mooring from Hamilton (2007) and the along-isobath velocity component (daily-averaged, 200 m above the bottom) is plotted in (C) which also shows the corresponding vector sticks. The dashed arrowed lines in the time-series plot in (C) indicate periods when bursts of TRW's were identified by Hamilton (2007). (Data courtesy of Dr. Peter Hamilton.) (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

for TRW's it is N_T. Fig. 2 illustrates the similarities and differences between IW's and TRW's.

Fig. 1B plots N_T in the vicinity of the Sigsbee escarpment in the Gulf of Mexico. The escarpment appears as the high-N_T region inside the 0.2 cpd contour, i.e. where TRW's with periods shorter than 5 days are trapped. Mathematically, this is equivalent to IW-trapping in a thermocline. It is helpful to think of the trapping region as an n-day 'TRW-valley' within which TRW's with periods shorter than the (contour of the) bounding period of n days are trapped (Fig. 2). Fig. 1B indicates that TRW's with periods shorter than 10 days propagating upslope onto the Sigsbee escarpment may be trapped, since beyond (i.e. north of) the escarpment there exist elevated ${N_T}^{-1}$ ("TRW-ridges") where the allowable periods are longer than 10 days; though in this case the waves may escape upslope through one of the narrow valleys. The existence of short-period TRW-valleys near the Sigsbee escarpment explains why predominantly shorter-period TRW's are found there (Lai and Huang, 2005; Hamilton, 2007; Oey, 2008).

The equivalence between TRW and IW-rays means that one may utilize tools well-developed for the latter (Lighthill, 1978) to analyze TRW's. There are some differences, but once they are understood, most of the analytical treatments for IW-rays carry over to TRW's. Firstly, as mentioned above, TRW's are confined only to the left half (for $f > 0$) of the equivalent xz-plane of IW's. Secondly, as seen in Fig. 1B, the n-day 'TRW-valleys' may be closed. With some exceptions,[1] the thermocline is unbounded for trapped IW's to propagate. Finally, whereas a trapped IW can reflect off side boundaries and therefore propagate back and forth ad infinitum within the thermocline (if inviscid), a trapped TRW cannot; the TRW phase, hence also its energy (irrespective of the wavelength, by Eq. (3)), is constrained to propagate with the shallower water on the right (in northern hemisphere). When there is more than one group of waves, the possibility exists that TRW's may focus at the "western" end of a TRW-valley.

In analogy with IW caustics (Lighthill, 1978), the phenomena when two or more TRW groups coalesce at a trapping boundary will be called topographic caustics or topocaustics. Near a trapping boundary, θ approaches $\pi/2$, and the $\mathbf{n}_{hy}$-component of the wavenumber and the corresponding group velocity approach zero (rays form $\mathbf{n}_{hy}$-cusps in the absence of a mean flow). Since the $\mathbf{n}_{hy}$-component of the wave energy flux (= energy density $\times$ group velocity) is constant, the energy (hence wave amplitude) itself is predicted to become infinite. The theory of caustics utilizes the fact when

[1] Side boundaries or bounding well-mixed regions with low N's.