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Open boundary conditions for internal gravity wave modelling using polarization relations

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ABSTRACT

This paper proposes an original approach of the open boundary condition problem, within the framework of internal hydrostatic wave theory. These boundary conditions are based on the relations of polarization of internal waves. The method is presented progressively, beginning with a simple case (non-rotating regime, propagation direction normal to the open boundary), ending with a more general situation (rotating regime, multimodal & multi-dimensional propagations and variable background field). In the non-rotating case and as far as we assume that the direction of propagation is locally normal to the open boundary, the so-called PRM (polarization relation method) scheme can be seen as a three-dimensional version of the barotropic Flather boundary conditions. The discrete form of the scheme is detailed. Numerical stability issues proper to leap-frog time stepping are in particular discussed. It is shown that errors on phase speed prescribed in the boundary conditions can notably deteriorate radiation properties. The normal mode approach is introduced to identify coherent structures of propagation and their corresponding phase speed. A simple and robust multi-dimensional propagation scheme can easily be derived from polarization relations. The rotating case is more difficult but it is possible, to some extent, to get around the dependency of phase speed on wave frequency and to keep the non-rotating formulation of the PRM conditions almost unchanged. The PRM scheme being applied to field anomalies, the question of the background reference state is addressed. The latter can be used to introduce incoming waves across the open boundaries or, alternatively, to represent the low-frequency variability of the model itself. The consistency of the pressure and tracer boundary conditions is finally discussed.

1. Introduction

There are basically two ways to prescribe the boundary conditions in regional and coastal oceanic circulation models: radiation boundary conditions or absorbing boundary conditions. The FRS method (Martinsen and Engedahl, 1987) is an absorbing condition frequently used by oceanic modellers and still subject to new investigations (Lavelle and Thacker, 2008). This method has interesting radiation properties, however, a particular attention should be paid to the reference state toward which the modelled inner solution is relaxed. Indeed, though it is numerically stable, the FRS condition will not produce a realistic behaviour in the case of an outgoing flux if the reference state does not match the modelled inner solution. In order to counteract this, several authors suggested to use a nudging layer with a large time constant (Marchesiello et al., 2001). In a way, this type of boundary condition is similar to the FRS condition. The difference between the FRS method of Martinsen and Engedahl (1987) which imposes a strong constraint on the boundary solution, and that of Jensen (1998) and Marchesiello et al. (2001), who imposes a weak constraint, is that the region near the boundary in the latter cases acts like a zone of nudging towards an imposed boundary condition. Since this kind of hybrid radiation-absorbing boundary condition seems to be widely used, using sponge or nudging layers does not exempt modellers from improving the performances of radiation conditions.

In free surface ocean models, internal and external modes are generally computed separately (Blumberg and Mellor, 1987; Shchepetkin and McWilliams, 2005). Authors, therefore, distinguish between the boundary conditions schemes for the external mode (barotropic conditions) and for the internal mode (3D or baroclinic conditions). The Flather radiation boundary condition (Flather, 1976), or characteristics condition (for a detailed description, see Blayo and Debreu, 2005) is frequently used for the external mode. Beside its interesting stability and conservation properties (Marsaleix et al., 2006), it is indeed a way to include efficiently a forcing term, and is consequently particularly appropriate for tide simulations (Oey and Chen, 1992).

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When applied to the internal mode, the efficiency of such radiation conditions is less obvious, and, within the framework of baroclinic studies, absorbing boundary conditions were proposed as an alternative (Carter and Merrifield, 2007). One of the main reasons for that is related to the fact that internal waves phase speed is difficult to estimate precisely, whereas surface waves phase speed is unambiguous ($c = \sqrt{gh}$) under the hydrostatic assumption. The internal waves phase speed depends not only on the stratification but also on the propagation mode. Moreover, the phase speed associated with a given baroclinic mode is related to the signal frequency. Finally, the possible mode coupling induced by the bathymetric slopes limits anyway the thrust of the modal approach in terms of the propagation speed. Orlanski (1976) proposed a method based on the analysis of the solution near the open boundaries in order to compute a local and time-evolving value for the wave phase speed. However, further studies demonstrated the weakness of the performances (Palma and Matano, 1998) as well as the unstable nature (Treguier et al., 2001) of the Orlanski type methods. Consequently, radiation conditions based on a fixed phase speed, a-priori representative of the main propagation mode (the first baroclinic mode), may appear as a reasonable option (Kourafalou et al., 1996).

Whether the phase speed is defined a-priori or computed using the Orlanski method, the radiation conditions for the internal mode are usually based on a Sommerfeld type wave equation. Although Nycander et al. (2008) recently proposed a set of characteristic conditions for the barotropic and baroclinic modes, studies dealing with a three-dimensional version of the barotropic Flather condition are, to our knowledge, rather scarce. This may be due to several technical problems, the resolution of which constitutes the scope of our study.

We actually propose a set of open boundary conditions based on the relations of polarization of internal waves. This approach has some similarities with the upper boundary condition for atmospheric limited-height models proposed by Bougeault (1983) and Klemp and Durran (1983). The method is presented progressively. We start with a simple case, that is, a single wave propagating in a non-rotating ocean in the direction normal to the open boundary (Section 2). The stability of different possible numerical schemes is discussed in Section 3.

More general cases are then addressed. It is known that the dispersive nature of internal waves is barely compatible with simplistic hypothesis on phase speed, generally found in usual 3D boundary schemes (Bennett and Chua, 1994). This problem can be partly solved by higher order schemes (Higdon, 1994) but we note that Nycander and Döös (2003) recommended against using the second-order Higdon's condition. As discussed in Section 4, the modal approach (Jensen, 1998, 1993) is a reliable alternative.

Currently used radiation conditions assume that the waves propagate in a direction normal to the open boundaries and are therefore not relevant for realistic multidirectional studies. Approximate boundary conditions for multidirectional propagations have been proposed by Engquist and Mayda (1977). Raymond and Kuo (1984) developed a method based on a Sommerfeld condition for multidirectional cases, however, several authors (Marchesiello et al., 2001; Barnier et al., 1998) questioned its stability in some cases. A stable method aimed at adapting Flather condition to three-dimensional cases is presented in Section 5. Finally, the definition of the reference state can also be an obstacle when adapting Flather condition to three-dimensional cases. The impact of this question is double: it concerns the potentially active nature of the boundary condition in the case of incoming flux, and can be a source of errors in the case of outgoing flux if the low-frequency evolution of the ambient stratification is not correctly taken into account. Those two points are detailed in Section 6. Numerical tests are performed in order to estimate the performances of the 3D Flather condition developed in this paper. The limitations of this method, in particular those induced by the rotating case, are discussed in Section 7. The consistency of pressure and tracer boundary conditions is finally discussed in Section 8.

2. Basic schemes

2.1. The non-rotating case

In this study, we use the regional and coastal oceanic circulation model SYMPHONIE. This model, fully described in Marsaleix et al. (2008), uses the Boussinesq and hydrostatic approximations. In the first case, and for the sake of clarity, we use a simple configuration where propagation is in the *Oxz* vertical plane. Practically, this means that we neglect hereafter the Coriolis term (note that the latter will be considered in Section 7) as well as the derivative with respect to y in the model equations.

First, we consider the equation proposed by Gill (1982, Section 6.4):

$$\frac{\partial u'}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x} \tag{1}$$

where (u', p') are the horizontal current and pressure perturbations associated with the wave. The total current and pressure are $(u, p) = (u' + u_{ref}, p' + p_{ref})$ where (u_{ref}, p_{ref}) corresponds to an ambient reference state evolving at a much smaller frequency than the perturbations. Then we consider a simple gravity wave propagating in the positive x direction:

$$(u', p') = (u_0, p_0) \times \cos(\omega t - kx) \tag{2}$$

This wave must leave the domain through the open boundary located downstream the propagation, where we propose to apply a boundary condition that satisfies (1) and (2):

$$p' = \rho_0\, cu' \tag{3}$$

where $c = \omega/k$ is the phase speed. A similar reasoning applied to the waves propagating in the negative x direction provides the boundary condition at the other open boundary: $p' = -\rho_0 c\, u'$. Replacing p' by the barotropic pressure $g\rho_0\eta$, u' by $\bar{u}$ and c by the surface wave phase speed in the long wave approximation, $\sqrt{gh}$, where h, η and $\bar{u}$ correspond, respectively, to the bathymetry, the free surface elevation and to the mean transport, this condition is analogous to the classical Flather barotropic condition, $\bar{u} \pm (g/h)^{1/2}\eta = 0$ (Johns et al., 1983) with no relaxation term. Eq. (3) can also be seen as a simple deduction of the relation of polarization proposed by Gill (1982, p. 263, Eq. 8.5.3). In the following, this scheme will be referred as a polarization relation method (PRM).

We note that u_{ref} is not necessarily negligible compared to typical phase speeds c and thus Eq. (1) could have included an advection term involving the reference field, namely $\frac{\partial u'}{\partial t} = -u_{ref}\frac{\partial u'}{\partial x} - \frac{1}{\rho_0}\frac{\partial p'}{\partial x}$ (Morel et al., 2008). This would have led to replacing the boundary condition (3) by $p' = \rho_0(c + u_{ref})u'$. However, we did not retain this formulation, as it is not compatible with the normal modes approach proposed in Section 4, where it is presumed that only p' and u' depend on z. Thus, using the PRM method requires that $u_{ref} \ll c$ to be valid.

2.2. Numerical implementation

The ocean model is based on a C-grid (Arakawa and Lamb, 1977). Fig. 1 shows the discrete distribution of the variables near the open boundary involved in the boundary condition defined by Eq. (3). In anticipation of the following sections dealing with fully three-dimensional cases, Fig. 1 also indicates the position of