Ocean Modelling 29 (2009) 85–93

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

Inverse estimation of open boundary conditions in tidal channels

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ARTICLE INFO

Article history:
Received 7 October 2008
Received in revised form 23 January 2009
Accepted 4 March 2009
Available online 14 March 2009

Keywords:
Tidal channels
Boundary conditions estimation and prediction
Quadratic programming
Lagrangian data assimilation

ABSTRACT

This article presents a novel algorithm for the estimation of open boundary conditions in river systems where tidal forcing is present. This algorithm uses a linearisation of the model equations. With the help of a linear discretisation scheme, the article presents a quadratic programming formulation of the estimation problem in which the control variables are the coefficients of the dominant tidal modes. This method is implemented for a scenario in which only Lagrangian observations from drifters are available to measure flow quantities. The performance of the algorithm is evaluated using numerical experiments and comparing estimation results with boundary conditions from a river located in the Sacramento Delta. The sensitivity of the algorithm to the number of modes estimated and its predictive capabilities are also assessed.

1. Introduction

The study of tidal forcing in bays and estuaries is crucial to the monitoring of water quality issues in these areas (Fischer et al., 1979; Yi et al., 1989). Numerically, these flows can be simulated using a full three-dimensional approach, or the one- or two-dimensional shallow water equations (Chadwick et al., 2004). In any case, the numerical model needs to be thoroughly calibrated with parameters depending on geometry and flow features. In addition, a knowledge of open boundary conditions is required for the model to perform adequately. Such boundary conditions can be inferred from measurements made with fixed sensors placed at the boundaries of the domain of interest or as is developed in this article, using drifters that are circulating in the system under consideration; it is also possible to extend the computational domain beyond the boundaries of interest but knowledge of the flow properties at a boundary is still required. Data assimilation techniques can be used to incorporate these observations into the model; they originated several decades ago in meteorology and oceanography (Anthes, 1974; Le Dimet and Talagrand, 1986; Sasaki et al., 1955). A number of different methods have been introduced over the years, which include variational methods (Navon, 1998, 1985), ensemble Kalman filtering (Evensen, 2007; Kuznetsov et al., 2003), optimal statistical interpolation (Molcard et al., 2003), or the nudging method (Ishikawa et al., 1996; Paniconi et al., 2003). The topic of inverse estimation of boundary conditions in meteorology and oceanography has been studied over the past few decades starting with the pioneering work (Sasaki et al., 1955) and more recently (Navon, 1985; Shulman et al., 1998; Yang and Hamrick, 2005; Yi et al., 1989; Zhang et al., 2003). In this article, a novel quadratic programming based variational data assimilation algorithm will be applied to the estimation of open boundary conditions for tidal channel flows.

The need for boundary condition estimation is driven in our case by operational requirements: we are interested in deploying drifter fleets in specific areas of the Sacramento-San Joaquin Delta for which current sensing infrastructure and modelling capabilities are insufficient. The Sacramento-San Joaquin Delta is at risk of extensive levee failures, which could be caused by earthquakes, flood or human activities. In such an event, the water quality of the Delta will be negatively affected, due to sediment suspension, salinity intrusion, and potentially agricultural contaminants. It is critical that transport models be available for use following such an event, even in the case where the entire geometry of the system has been altered. Unfortunately, existing models of the Sacramento-San Joaquin Delta rely heavily on historical data sets for calibration. These models would be of limited utility if the system were radically altered, as would occur in the case of extensive levee failures. The use of rapidly deployed Lagrangian drifters and the estimation of boundary conditions is motivated by this need: we aim to develop a sensing-modelling system that is capable of predicting regional flows and transport in the Delta in real-time without dependence on historical data. The timescale of interest for this analysis is on the timescale of days: we would like to be able to project transport patterns forwards in time by a few days based on only a few days of data.

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doi:10.1016/j.ocemod.2009.03.002

The immediate objective is to estimate open boundary conditions for a model of flows in tidal channels of the Sacramento-San Joaquin Delta. To this end, we develop variational data assimilation techniques in which a cost function measuring the norm of the difference between observations gathered by drifters and model predictions is minimised, subject to constraints given by the discretised model equations; the control variables are the coefficients of the dominant tidal modes present in the upstream boundary condition. A similar variational approach for Lagrangian data assimilation in rivers applied to bottom topography estimation was presented in (Honnorat et al., 2009). The estimation of boundary conditions in the Sacramento-San Joaquin Delta using fixed sensors and data reconciliation techniques was presented in (Wu et al., 2009) which presents some similarities with our work, but specifically tackles the problem of Eulerian measurements. While a two-dimensional model could be chosen for the identification of the boundary conditions, its computational cost is high and can be avoided by using simpler models such as a one-dimensional model. The choice we make here is motivated by the desire to have a rapid, robust estimate of boundary conditions that can be used in real-time flow simulations. A somewhat similar approach was used in (Gejadze and Monnier, 2007) as a one-dimensional shallow water model was combined with a local two-dimensional model through optimal control methods. In (Gejadze and Copeland, 2006; Gejadze et al., 2006), a method for the estimation of open boundary conditions for the Navier–Stokes equations with a free surface from fixed depth and velocity measurements is developed. All the articles mentioned above use the adjoint method combined with a quasi Newton solver to solve a variational problem. The adjoint method has the main drawback of a high computational cost as 50–100 iterations are usually required which corresponds to 100– 200 numerical resolutions of the direct and adjoint equations. Additionally, the nonlinearity of the problem means that convergence to a global minimum is not guaranteed. In this article, we use a quadratic programming approach which eliminates both issues. Indeed, since the cost function is quadratic, this problem can be solved as a quadratic program provided the constraints are linear. Another novelty in this article is the use of drifters for the estimation of boundary conditions while the other articles previously mentioned rely on fixed measurement stations. Note also that unlike adjoint based optimisation, the quadratic programming technique used in the present article does not require the definition and resolution of an adjoint (backward) problem.

This article is organised as follows: we start by presenting the basics of tidal channels and we state the problem solved in the article; we then use standard data assimilation terminology to develop the estimation algorithm; numerical experiment settings are then described before the results for estimation and prediction of tidal flow are presented and analysed.

2. Model

2.1. Tidal channels

Tidal channels are bodies of water in which periodic changes in the water level and velocity field occur under the influence of tides, along the dominant tidal frequencies such as the K1 tide generated by the Sun, the M2 tide generated by the Moon or the MK3 shallow water tide created by the nonlinear interaction between the K1 and M2 tides resulting from the bottom friction. In channel networks, tidal trapping (Fischer et al., 1979) can occur due to phase effects, with the flow directions changing at different times in the deeper and shallower branches; for example, the flow may be ebbing in a shallow channel while a nearby channel is already flooding. The implications for transport and dispersion are profound and tidal trapping is likely the dominant dispersion observed in such systems.

The flow in such a channel can be modelled using two-dimensional depth-averaged shallow water equations such as (Vreugdenhil, 1994):

$$\frac{\partial h}{\partial t} + \vec{u} \cdot \nabla h + h \nabla \cdot \vec{u} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \nabla u = -g \frac{\partial h}{\partial x} - g \frac{\partial b}{\partial x} + f_x \tag{2}$$

$$\frac{\partial v}{\partial t} + \vec{u} \cdot \nabla v = -g \frac{\partial h}{\partial y} - g \frac{\partial b}{\partial y} + f_y \tag{3}$$

where (u, v, h) represent the velocity components and water depth, with $\vec{u} = (u, v)$, b the bottom elevation, g the acceleration of gravity and (f_x, f_y) the viscous forces term.

For the simulations presented in this article, we will use a specific realisation of (1)–(3) incorporating friction forces and a specific turbulence model:

$$\frac{\partial h}{\partial t} + \vec{u} \cdot \nabla h + h \nabla \cdot \vec{u} = 0 \tag{4}$$

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \nabla u = -g \frac{\partial h}{\partial x} - g \frac{\partial b}{\partial x} + F_x + \frac{1}{h} \nabla \cdot (h \nu_t \nabla u) \tag{5}$$

$$\frac{\partial v}{\partial t} + \vec{u} \cdot \nabla v = -g \frac{\partial h}{\partial y} - g \frac{\partial b}{\partial y} + F_y + \frac{1}{h} \nabla \cdot (h \nu_t \nabla v) \tag{6}$$

The friction forces are given by the following Manning law:

$$F_x = -\frac{1}{\cos \alpha} \frac{g m^2}{h^{4/3}} u \sqrt{u^2 + v^2} \tag{7}$$

$$F_y = -\frac{1}{\cos \alpha} \frac{g m^2}{h^{4/3}} v \sqrt{u^2 + v^2} \tag{8}$$

where h is the total depth of water, $\vec{u} = (u, v)$ is the velocity field, g is the acceleration of gravity, b is the bottom elevation, ν_t is the coefficient of turbulence diffusion obeying the so-called k-epsilon model (see (Rastogi and Rodi, 1978) for more details), $\alpha = \alpha(x, y)$ is the slope of the bottom, and m is the Manning coefficient (usually denoted by n). In the present case, the Manning coefficient is chosen to be constant in time and space and equal to 0.02, corresponding to a highly frictional bottom. Finally, t is time and x, y are horizontal space coordinates.

2.2. Problem description

An accurate simulation of tidal trapping relies on a prediction of the phase differences which is usually estimated according to historical data sets. In this article, our goal is to estimate open boundary conditions, more precisely the amplitudes and phases corresponding to the main four to eight tidal modes using velocity and position measurements provided by a number of drifters which are released on a portion of the Sacramento River located in the Sacramento-San Joaquin Delta in California. While the model used for the data assimilation problem is two-dimensional, the estimation of the boundary conditions is performed on a one-dimensional model which yields substantial benefits with respect to the size of the problem solved and therefore the computational time. The use of a one-dimensional estimation of the open boundary conditions is justified by the rapid lateral adjustment time of the free surface (< 1 min); further, the velocity profile can be reconstructed using interpolation from the average velocity in the cross-section. For example, in the case of the software package TELEMAC used in the article, the velocity profile is reconstructed with the assumption that the velocity is proportional to the square root of the depth along the cross-section (Hervouet and Haren, 2002).

The algorithm proposed by this article consists in minimising the difference between measured velocity at the location of the