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A one-dimensional benchmark for the propagation of Poincaré waves

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Abstract

Several numerical methods are employed to solve the linear shallow-water equations describing the propagation of Poincaré waves within a one-dimensional finite domain. An analytical solution to the problem, set off by a discontinuous steplike elevation, is known and allows for assessing the accuracy and robustness of each method and in particular their ability to capture the traveling discontinuities without generating spurious oscillations. The following methods are implemented: the method of characteristics, the Galerkin finite-element method (FEM) and the discontinuous Galerkin FEM with two different ways of computing the numerical fluxes.

Keywords: Poincaré waves; Method of characteristics; Discontinuous finite elements; Riemann solver

1. Introduction

Motion in the ocean spans a very wide range of timescales. While the large-scale circulation is characterized by velocities on the order of up to one meter per second and timescales that can be as large as hundreds of years, the fast-propagating inertia–gravity waves exhibit phase velocities on the order of hundreds of meters per second and much smaller timescales. Internal gravity waves propagate with velocities on the order of one meter per second or less. The vast disparity of ocean processes timescales poses a challenge in numerical ocean modeling. If an explicit time step is used, it is limited by the so-called

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Courant–Friedrichs–Lewy (CFL) condition, which states that the time step should not be larger than the travel time of the fastest physical process over the smallest space increment. In free surface ocean models that allow for the existence of external inertia–gravity (Poincaré) waves, the upper bound on the time step is far smaller than more practical time steps that would permit time integration over thousands of years on today's computers. The first attempt at circumventing this problem by replacing the free surface by a rigid lid—thereby eliminating external inertia–gravity waves—has been widely dismissed. Among the rationales for such a design are that a rigid lid distorts the properties of large-scale barotropic Rossby waves, does not permit tidal modeling and complicates inclusion of fresh water flux surface boundary condition (Killworth et al., 1991; Dukowicz and Smith, 1994; Deleersnijder and Campin, 1995; Hallberg, 1997; Higdon and de Szoeke, 1997).

A common alternative no longer relies on the rigid-lid approximation. The ocean surface is free and remains a prognostic variable but the governing equations are split into subsystems that model the fast and slow motions separately. These subsystems are generally referred to as the barotropic and baroclinic systems, respectively, or the external and internal modes, respectively. Fast motions are approximately independent of the vertical coordinate z so that the external mode is two-dimensional and is well represented by the shallow-water equations that model the motion of fluid layers of constant density. Slow motions are fully three-dimensional, however, but the restriction on the time step is dictated by the internal dynamics, of which timescales are several orders of magnitude larger than that of the external mode. The latter can be solved explicitely with small time steps or implicitely with larger time steps. Choosing an implicit treatment eliminates the constraint imposed by the CFL condition but leads to large systems to be solved at each time step. This choice can be made for tidal and tsunami calculations provided that a reduced time step be used. If an explicit approach is considered for the barotropic mode, the number of small barotropic time steps for each large baroclinic time step is roughly the ratio of barotropic inertia–gravity wave speed to baroclinic internal gravity wave speed (Killworth et al., 1991). Details on mode splitting implementations can be found in Blumberg and Mellor (1987), Hallberg (1997), Higdon and de Szoeke (1997) and Higdon (2002).

Large-scale oceanic motions roughly obey the geostrophic equilibrium. When imbalances occur, the geostrophic balance is restored by means of Poincaré waves. In strongly stratified seas, internal inertia–gravity waves are generated when displacement of density surfaces occurs. Those waves respond to the same physical mechanism as external Poincaré waves (Gill, 1982). In models allowing for the existence of inertia–gravity waves, it is of paramount importance to represent those waves accurately. In that respect, the coupled issues of time and space discretization ought to be focused on. Time stepping is not the subject of this paper (see e.g., Beckers and Deleersnijder, 1993) as we concentrate on spatial discretization. A one-dimensional benchmark for the propagation of Poincaré waves is proposed. This problem bears many similarities with the classical geostrophic adjustment initially studied by Rossby and further investigated by Gill (1976) for the linear part and Kuo and Polvani (1996) for its nonlinear counterpart. In this paper, the linearized shallow-water equations, in which homogeneity is assumed in the y-direction, are solved in a domain of finite length with an initial discontinuous elevation field. The design difference with adjustment problems lies in the finiteness of the domain in the x-direction. Whereas in adjustment problems, an infinite domain in the x-direction is considered, we study the case of Poincaré waves propagation in a finite domain. In so doing, no end state is ever reached and, in the absence of friction, wave propagation goes on forever within the domain. The persistence of the discontinuities is the prominent feature of the time-dependent solution presented by Gill (1976). It also appears in the solution to our benchmark, thereby posing a challenge for classical numerical methods to solve the problem. A numerical method will be appraised based upon its ability to capture the traveling discontinuity without generating spurious oscillations. The following methods are considered in this paper: the method of characteristics, the Galerkin finite-element method (FEM) and the discontinuous Galerkin FEM with two different ways of computing the numerical fluxes.

2. A one-dimensional benchmark

The linearized governing equations for a single, inviscid, homogeneous shallow layer of fluid on an f-plane are the shallow-water equations, given by