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Coriolis weighting on unstructured staggered grids

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Abstract

There is an increasing interest to move ocean codes from classical Cartesian staggered mesh schemes to unstructured staggered grids. By using unstructured grid models one may construct meshes that follow the coastlines more accurately, and it is easy to apply a finer resolution in areas of special interest.

In this paper we focus on how to approximate the Coriolis terms in such unstructured staggered grid models using equivalents of the Arakawa C-grid for the linear equations governing the propagation of the inertia-gravity waves. We base the analysis on a Delaunay triangulation of the region in question and use the Voronoi points and the midpoints on the triangle edges to define a staggered grid for the sea elevation and the velocity orthogonal to the edges of the triangles. It is shown that a standard method for the Coriolis weighting may create unphysical growth of the numerical solutions. A modified Coriolis weighting that conserves the total energy is suggested.

In real applications diffusion is often introduced both for physical reasons, but often also in order to stabilise the numerical experiments. The growing modes associated with the unstructured staggered grids and equal weighting may force us to enhance the diffusion more than we would like from physical considerations. The modified weighting offers a simple solution to this problem.

Keywords: Shallow water equations; Unstructured grid; Staggered mesh; Numerical stability; Coriolis; Energy

1. Introduction

Even if numerical ocean models have proved to be very useful in many oceanic studies, the limitations of models based on structured grids have become apparent. There is a need to resolve both the large scale flow over larger domains and the small scale flow in more targeted areas. By using ocean models based on unstructured meshes one may apply coarser resolution over most of the computational domain and a gradually finer resolution towards the areas of special interest, allowing the resolution of both the larger scale and the smaller scale flow in these areas. Important work on the building of unstructured mesh ocean models is reviewed recently in Pain et al. (2005), and the advantages and the current status in the field are discussed in this paper. Finite element models have been developed and applied in many studies, see for instance Lynch and Werner

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(1987, 1991), Lynch et al. (1995), and recently non-hydrostatic finite element models for oceanic flow are presented, see Ford et al. (2004a,b) and Labeur and Pietrzak (2005). Also finite difference and finite volume models based on unstructured grids have been developed (Casulli and Walters, 2000; Ham et al., 2005). The unstructured grids have many advantages. However, there may be some problems with the waves propagating from areas with a fine resolution to areas with a coarser resolution, see for instance Hall and Davies (2005). The stability of the numerical solutions may be affected when focusing the grids towards the targeted areas. Furthermore, there may still be un-revealed weaknesses with the unstructured grid approach. Based on this, and on the numerical results we have seen so far, it is not yet evident that unstructured grid models for the ocean produce better model results than the structured grid models.

The quality of the unstructured mesh is important, and parallel to the development of unstructured mesh ocean models, there is a growing literature on the mesh generation, see Castro-Diaz et al. (1997), Pain et al. (2005) and Gorman et al. (2006). Particularly there is a great interest in the Delaunay mesh generation, see Borouchaki and Lo (1995), Borouchaki et al. (1996), Xu et al. (1998), Legrand et al. (2000), Shewchuk (1996, 2002) and Persson and Strang (2004).

Ocean models often define the variables on staggered grids. Many regional scale models apply the C-grid (Mesinger and Arakawa, 1976) because of its satisfactory properties provided that the grid resolution is high relative to the deformation radius, see Mesinger and Arakawa (1976), Fox-Rabinowitz (1991), and/or Arakawa and Lamb (1977). In the unstructured grid world they also have to choose the positioning of the velocity points and the pressure points in the grid elements. Equivalents of the Arakawa's A-, B-, and C-grid are investigated in some recent papers on the linear shallow water equations, see Le Roux et al. (1998, 2005) and Hanert et al. (2002).

The stability properties of the numerical methods are often studied with the Fourier or the von Neumann method. This method is relatively easy to apply to small subsystems of equations, but gives only necessary conditions for the stability. The method is only applicable to linearised equations and constant parameters i.e. constant depth of the ocean. See for instance Grammeltvedt (1969), Mesinger and Arakawa (1976), Schoenstadt (1977), Foreman (1984), Tanguay and Robert (1986) and Fox-Rabinowitz (1991) or the recent study by Wang (1996) where a theorem of roots for some polynomials is applied to extend previous studies to more complex systems. In Le Roux et al. (1998) different types of low-order finite elements are analysed using this method.

When applying the energy method, one requires that invariants associated with the continuous equations, the total energy, the vorticity and/or the enstrophy, are conserved in the discrete representation of the integral constraints. This method may be applied to non-linear systems to produce sufficient conditions for the stability. The task is to construct discrete spatial operators such that the contributions to the discrete invariants cancel when summarised over the model region. This is often done in the space domain, see Haltiner and Williams (1980), Arakawa (1966), Arakawa and Lamb (1981), and Salmon (2004). The conservative schemes constructed using the energy method become rather complicated involving large computational stencils and may therefore be difficult to use in practice. Lilly (1965) also applied the technique to the vorticity equation studying the invariants in the discrete Fourier domain.

When propagating the solution of the partial differential equations describing geophysical fluid flow with numerical models, instabilities may often occur. In particular steep topography often in combination with non-linearities may cause an energy cascade towards the shortest resolvable wavelengths of the computational grid, see Arakawa and Lamb (1981) and Adcroft et al. (1999). Espelid and Berntsen (1997) focus on the stability using the C-grid centered differences in space and using different numerical time-stepping methods on problems with varying depth. It is demonstrated that variable depth may cause instabilities that may not be removed by reducing the time step. In a recent paper Espelid et al. (2000) show that it is possible to avoid this problem by a proper Coriolis weighting that ensures that the approximated model achieves a local conservation of the energy. The suggested weighting takes into account both varying depth and a possible variation of the Coriolis parameter over the region.

In this paper we want to generalise this idea to unstructured staggered grids when applied to ocean modelling. Unstructured grids offers a more flexible way to model these problems but will at the same time introduce an additional set of parameters that may vary over the region and thus influence how one should do the Coriolis approximation. We will in this paper focus on a triangular mesh and assume that the region consists