Available online at www.sciencedirect.com

SCIENCE DIRECT®

Ocean Modelling 13 (2006) 44–64

Ocean Modelling

www.elsevier.com/locate/ocemod

Numerical error assessment and a temporal horizon for internal waves in a hydrostatic model

Ben R. Hodges [a,*], Bernard Laval [b,1], Bridget M. Wadzuk [c,2]

[a] *Department of Civil, Architectural and Environmental Engineering, University of Texas at Austin, 1 University Station, C1786, Austin, TX 78712, USA*

[b] *Department of Civil Engineering, University of British Columbia, 6250 Applied Sciences Lane, Vancouver, BC, Canada V6T 1Z4*

[c] *Department of Civil and Environmental Engineering, Villanova University, 800 Lancaster Ave., Villanova, PA 19085, USA*

Received 20 April 2005; received in revised form 26 September 2005; accepted 28 September 2005

Abstract

Three-dimensional hydrostatic models are often used to represent stratified dynamics and large-scale circulations in lakes and coastal oceans. However, non-linear internal wave evolution is fundamentally non-hydrostatic, so any hydrostatic model will eventually fail if non-hydrostatic effects accumulate. In this paper, we demonstrate that the time scale of baroclinic wave steepening provides a temporal horizon for model skill when sufficiently fine model grid scales are applied. At coarser grid scales, model results may be dominated by either diffusion or dissipation prior to the time scale of steepening. Methods for estimating numerical viscosity and diffusivity are developed, along with approximate predictive scales for the Centre for Water Research Estuary and Lake Computer Model (CWR-ELCOM). Results indicate that numerical diffusion of the pycnocline (i.e. thermocline or halocline) may rapidly increase past the time scale of steepening for a well-resolved model, and that somewhat coarser horizontal grid scales may be preferable for integrations beyond the time scale of steepening. While the present application and analyses are for a specific model, the methodology is developed in a general formulation and could be applied to other models for insight into error accumulation.

Keywords: Internal wave; Hydrostatic model; Model error; Baroclinic; Coastal ocean

---

1. Introduction

Based on practical computational limits and justified by a priori scaling analyses, three-dimensional (3D) models used for lakes, estuaries and coastal oceans typically apply the hydrostatic approximation, which is reasonable for large-scale, predominantly horizontal phenomena (e.g. Kantha and Clayson, 2000). Furthermore,

---

[*] Corresponding author. Tel.: +1 512 471 4730; fax: +1 512 471 5870.

E-mail addresses: hodges@mail.utexas.edu (B.R. Hodges), blaval@civil.ubc.ca (B. Laval), bridget.wadzuk@villanova.edu (B.M. Wadzuk).

[1] Tel.: +1 604 822 2204; fax: +1 604 822 6901.

[2] Tel.: +1 610 519 5365; fax: +1 610 519 6754.

1463-5003/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ocemod.2005.09.005

simple application of non-hydrostatic models at practical horizontal grid scales is unlikely to achieve any significant improvements in model skill in the near future; correctly modeling non-hydrostatic processes depends on horizontal model grid scales that represent the non-hydrostatic scales of motion (which are similar to the vertical scales). Unless horizontal grid scales are on the order of 1/10th of the water depth, we consider it unlikely that a 3D non-hydrostatic model will resolve the non-hydrostatic phenomena. However, the hydrostatic approximation is expected to fail only in localized regions where the vertical and horizontal scales are of similar magnitude, so the utility of the hydrostatic approximation for coastal ocean (and similar) modeling seems secure for the near future.

Coastal ocean, estuarine and lake models must contend with a wide range of forcings and physics that place competing demands on model resolution for a given computational capability. As compromises are always somewhat unsatisfactory, modelers have welcomed the continuing increases in computer power that allow improved grid resolution and representation of a more detailed range of physics. Hydrostatic models in 3D (e.g. Hodges et al., 2000; Blumberg and Mellor, 1987; Haidvogel et al., 2000) are now routinely used for large-scale stratified systems at spatial and temporal resolutions that allow internal waves to be represented. Being able to capture the physics of internal waves is considered valuable as non-linear internal waves readily steepen and break along sloping boundaries (Boegman et al., 2003; Kao et al., 1985), which impacts mixing across the pycnocline, sediment resuspension, and the distribution of both phytoplankton and nutrients in the water column (De Silva et al., 1997; Nishri et al., 2000). It follows that a model's predictive ability for both physical and ecological processes may be affected by its skill in representing internal wave evolution.

Unfortunately, the real-world non-linear evolution of internal waves is inherently non-hydrostatic (e.g. Long, 1972; Whitham, 1974; Lighthill, 1978). The non-hydrostatic pressure gradient associated with wave motion opposes non-linear wave steepening. Solitary waves are formed in the process of non-linear steepening when steepening and non-hydrostatic dispersion effects come into balance (cf. Kinsman, 1984; Miropol'sky, 2001); the resulting wave (or wave train) may propagate indefinitely (in the inviscid limit) without changing form. To draw these observations back to the hydrostatic approximation, large-scale horizontal barotropic and baroclinic pressure gradients will typically force large-scale flows (associated with coastal oceans, estuaries and lakes). Internal wave non-hydrostatic pressure gradients are locally strong, but are small-scale so the non-hydrostatic pressure gradient can often be neglected without seriously affecting the large-scale model results (Kantha and Clayson, 2000). However, neglect of non-hydrostatic effects while retaining non-linear effects is arguably relevant to model performance because of directional biases: non-linearity always acts to steepen an internal wave and is in opposition to both dispersive (non-hydrostatic) and viscous forces. Thus, by neglecting the dispersive processes governing internal wave evolution while leaving the steepening processes intact, the hydrostatic approximation introduces a bias that favors wave steepening (e.g. Daily and Imberger, 2003). Regardless of a modeling effort's focus, the occurrence of internal waves in a simulation may affect model skill depending upon how non-linear steepening affects numerical error.

In this paper, we demonstrate that the hydrostatic approximation leads to a temporal horizon for internal waves that limits model skill in their representation and cannot be overcome by improved grid resolution. Beyond this time horizon, improved grid refinement simply provides a better solution to the wrong equations. It follows that the underlying validity of a hydrostatic model for a complex coastal ocean system may not be controlled by the accuracy of the numerical method, but instead may be limited by accumulation of numerical error from hydrostatic modeling of internal waves. Researchers focused on wave phenomena might call this observation trivial as the use of hydrostatic models is, strictly speaking, inappropriate for such non-hydrostatic phenomena. However, internal waves are only one of many phenomena of interest in the coastal ocean, and are only one of the myriad of reasons for applying finer grid scales to a problem. Thus, modelers must contend with models that may be representing physical phenomena (internal waves) in a non-physical way (hydrostatically). To provide further insight on this problem, we present methods for estimating numerical dissipation and diffusion in a hydrostatic model of internal waves. We further demonstrate how this method can be used to develop approximate predictive scalings for numerical diffusivity and viscosity as a function of grid resolution and internal wave characteristics. It presently appears that such scalings cannot be developed a priori from numerical theory. Indeed, despite recent advances in verification methods (Roy, 2003), it is not clear that the effects of non-linear terms and the coupling between scalar and momentum transport can be discussed in any general way given the complexity of mixed-order numerical discretization methods applied in a