Computers and Electrical Engineering 41 (2015) 1–8

Contents lists available at ScienceDirect

Computers and Electrical Engineering

journal homepage: www.elsevier.com/locate/compeleceng

Application of van der Pol–Duffing oscillator in weak signal detection ☆

Zhao Zhihong [a,b,*], Yang Shaopu [b]

[a] School of Computing and Informatics, Shijiazhuang Tiedao University, Shijiazhuang 050043, China
[b] Key Laboratory of Traffic Safety and Control in Hebei, Shijiazhuang 050043, China

ARTICLE INFO

Article history:
Received 4 April 2014
Received in revised form 17 November 2014
Accepted 17 November 2014
Available online 6 December 2014

Keywords:
Van der Pol–Duffing oscillator
Weak signal detection
Chaos
SNR

ABSTRACT

This study presents a new weak signal detection method based on the van der Pol–Duffing oscillator. The principle of the proposed method is described. A weak signal is detected through the transition from the chaotic to the periodic state. Numerical simulation shows that the van der Pol–Duffing oscillator is sensitive to a weak signal under strong noise conditions. Several aspects of the proposed method, including the noise influence, influence of different frequency signals, and influence of the phase shift, are studied in detail. Results indicate that the application of the van der Pol–Duffing oscillator to weak signal detection is feasible.

1. Introduction

Weak signal detection is a challenging task in signal detection and early machinery fault detection. Conventional detection methods, such as wavelet method, are limited under strong noise conditions. The chaotic oscillator is highly effective in weak signal detection because of its properties of noise immunity and sensitivity to specific frequency signals.

The Duffing oscillator is the most frequently used chaotic oscillator for detecting weak signals [1–5]. When the Duffing system is in a critical state, a small perturbation may result in system state changes. The weak signal is often detected through the transition from the chaotic to the periodic state. Although the Duffing oscillator is widely used for weak signal detection, further research is necessary to determine whether other chaotic oscillators are suitable for detecting weak signals [6].

The van der Pol–Duffing oscillator is employed to solve physical, engineering, and even biological problems. This oscillator type is a generalization of the classic van der Pol oscillator. The van der Pol–Duffing oscillator without an external force has a rich dynamic behavior [7,8], including chaos and bifurcation.

This study focuses on weak signal detection based on the van der Pol–Duffing oscillator. First, we analyze the dynamic behavior of the van der Pol–Duffing oscillator. Second, we realize weak signal detection according to the phase trajectory change of the van der Pol–Duffing oscillator from the chaos to the periodic state. Finally, some problems are discussed relative to weak signal detection by the van der Pol–Duffing oscillator. This study is the first to use the Van der Pol–Duffing oscillator for weak signal detection. Numerical simulation results show that the proposed method has satisfactory detecting performance.

☆ Reviews processed and recommended for publication to the Editor-in-Chief by Associate Editor Dr. Zheng-Hua Tan.

* Corresponding author.
E-mail address: hb_zhaozhihong@126.com (Z. Zhihong).

http://dx.doi.org/10.1016/j.compeleceng.2014.11.007
0045-7906/

The remainder of this paper is organized as follows: Section 2 introduces the van der Pol–Duffing oscillator. Section 3 describes the principle and implementation of weak signal detection using the van der Pol–Duffing oscillator. Section 4 presents the related aspects of the proposed method. Section 5 concludes.

2. van der Pol–Duffing oscillator

The van der Pol–Duffing oscillator is described by the following nonlinear equation [9,10]:

$$\ddot{x} - \mu(1 - x^2)\dot{x} + x + \alpha x^3 = F\cos(\omega t) \tag{1}$$

where μ and α are two positive coefficients; and F and ω are the amplitude and frequency of the external excitation, respectively. Eq. (1) is a generalization of the classic van der Pol oscillator equation.

The three equilibrium points of the system in Eq. (1) for $F = 0$ correspond to $x + \alpha x^3 = 0$, such that we have the fixed points $\left(0, -\sqrt{-\frac{1}{\alpha}}, \sqrt{-\frac{1}{\alpha}}\right)$.

Assuming $\dot{x} = y$, we then derive the following:

$$\begin{cases} \dot{x} = y \\ \dot{y} = \mu(1 - x^2)\dot{x} - x - \alpha x^3 + F\cos(\omega t) \end{cases} \tag{2}$$

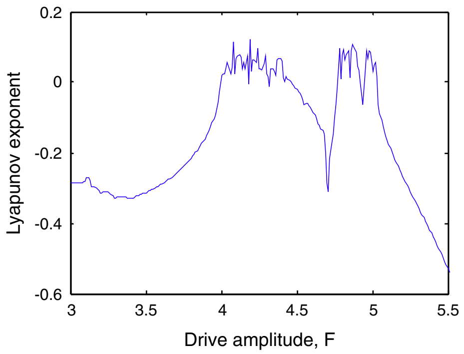

Fig. 1. Maximum Lyapunov exponent versus the amplitude F under the initial conditions of (0.1, 0.1).

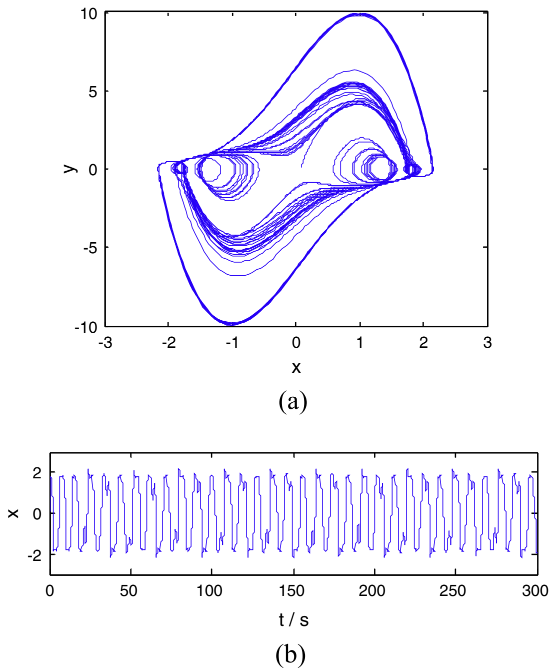

Fig. 2. (a) Phase plane diagram of chaotic motion with $F = 4.9$. (b) Time series diagram of chaotic motion.