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# Dynamical model of mesoscales in $z$-coordinates

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## Abstract

Using the equations of the dynamical mesoscale model developed previously [Ocean Modell. (2004) 8, 1–30], we derive a mesoscale model in $z$-coordinates to be used in coarse resolution OGCMs. We present a model for the eddy-induced velocities, mesoscale diffusivity, eddy kinetic energy, eddy potential energy, residual diapycnal flux, velocity across mean isopycnals, Reynolds stresses, E-P, PV and RV fluxes.

Specifically, in the *mean buoyancy equation*, mesoscales give rise to two terms: an eddy-induced velocity $\mathbf{u}_\mathrm{M} = (\mathbf{u}^+, w^+)$ and a residual diapycnal flux $\Sigma$. While $\mathbf{u}_\mathrm{M}$ has received much attention, $\Sigma$ has always been taken to be zero. Physical and numerical arguments are presented to show that $\Sigma$ is not zero. We present the model results for both $\mathbf{u}_\mathrm{M}$ and $\Sigma$. The new expression for $\mathbf{u}^+$ contains four terms, the first of which has the structure of the GM model while the remaining three terms are new. Several interpretations of the new terms are given. The boundary conditions at $z = -H, 0$ are satisfied by $\mathbf{u}_\mathrm{M}$ and $\Sigma$, thus avoiding the need for "tapering schemes" employed thus far to amend the failure of models for $\mathbf{u}_\mathrm{M}$ to satisfy the proper boundary conditions. We also present the model results for the mesoscale diffusivity and show that the predicted magnitude and $z$-dependence are in accord with recent numerical models. The residual flux $\Sigma$ gives rise to a mesoscale-induced diapycnal diffusivity which in the ACC is larger than the diabatic one. The resulting "velocity across mean isopycnals" may thus significantly affect the dynamics of the thermocline (e.g., the Munk–Wunsch advective–diffusive model). The predicted magnitude of this velocity is in an accord with recent results from eddy-resolving codes.

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The presence of a non-zero residual flux $\Sigma$ in the mean buoyancy equation implies that the effect of mesoscales in the T–S equations is not fully accounted for with only the eddy-induced velocity, as generally done. Additional $\Sigma$-like diapycnal fluxes must be added to the T–S equations.

*Mean momentum equations*. Since the down-gradient model used in most OGCMs does not represent mesoscales, the latter have not yet been accounted for. A model for the divergence of the Reynolds stresses, the Eliassen–Palm fluxes, the PV (potential vorticity) and RV (relative vorticity) fluxes is presented. We show that the Sverdrup vorticity balance is modified by mesoscales. In particular, while the standard Sverdrup relation does not allow meridional currents to cross the equator, the presence of mesoscales allows such a possibility.
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*Keywords:* Mesoscale modeling; Isopycnal and level coordinates; Bolus and eddy-induced velocity; $\Sigma$-term; Potential and relative vorticity fluxes

## 1. Introduction

A satisfactory representation of the very energetic, ~100 km size ocean mesoscale eddies in coarse resolution OGCMs (e.g., Griffies et al., 2000) is still an open problem. Significant progress was achieved when it was realized that mesoscale models had to reflect the fact that there is a huge reservoir of mean potential energy MPE (Lueck and Reid, 1984; Huang, 2004) which, through baroclinic instabilities, feeds mesoscales. It was therefore natural to model such process with a down-gradient expression to represent the transfer of energy from MPE to eddy potential energy EPE. The GM model (Gent and McWilliams, 1990, cited as GM) that was devised to explicitly capture this important physical process, considerably improved the performance of coarse resolution OGCMs (Gent and McWilliams, 1990; Danabasoglu et al., 1994; Böning et al., 1995; Hirst and McDougall, 1996, 1998). These improvements have rekindled the interest in mesoscale modeling and at the same time they revealed the complexity of the physical processes characterizing mesoscales.

It is fair to say that since the appearance of the GM model the bulk of the work is represented by numerical studies (both eddy resolving and coarse resolution ocean codes), as well as by heuristic models (Böning et al., 1995; Beckmann et al., 1994; Danabasoglu and McWilliams, 1995; Gille and Davis, 1999; Drijfhout and Hazeleger, 2001; McDougall and McIntosh, 2001; Karsten and Marshall, 2002; Radko and Marshall, 2004a,b; Ferreira and Marshall, submitted; Olbers and Visbeck, in press). While eddy-resolving codes have shed much led on the physics of mesoscales, it is difficult to translate such information into a model usable in OGCMs. At the same time, heuristic models cannot determine important parameters such as the mesoscale diffusivity and its $z$-dependence. In this context, it is of interest the suggestion by Muller and Garrett (2002) that any parameterization "must be specified as formulae rather than just numerical values". This requirement acquires particular relevance in the case of mesoscales since eddy fluxes at a given depth are functions of large scale fields not only at the same depth, but at all other depths (Muller and Garrett, 2002), a non-local feature that is quite hard to model phenomenologically, as models for the PBL (planetary boundary layer) have shown over the years (Cheng et al., 2002). Thus, there is a need to construct a mesoscale model based on the dynamic equations governing the mesoscales. In contrast to the rather large literature of numerical simulations and heuristic models,