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A convex regularization model for image restoration ☆

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ABSTRACT

Many variational formulations are introduced over the last few years to handle multiplicative data-dependent noise. Some of these models seek to minimize the Total Variation (TV) norm of the absolute gradient function subject to given constraints. Since the TV-norm (well-defined in the space of bounded variations (BVs)) minimization eventually results in the formation of piece-wise constant patches during the evolution process, the filtered output appears blocky . In this work the block effect (commonly known as staircase effect) is being handled by using a convex combination of TV and Tikhonov filters, which are defined in BV and $\mathcal{L}^2$ (square-integrable functions) spaces, respectively. The constraint for the minimizing functional is derived based on a maximum a posteriori (MAP) regularization approach, duly considering the noise distributions. Therefore, this model is capable of denoising speckled images, whose intensity is Gamma distributed. The results are demonstrated both in terms of visual and quantitative measures.

1. Introduction

Restoration of images under additive data-independent Gaussian noise is a well studied problem in image processing. Many methods are suggested in last two decades to address such a degradation to a considerable extent. Among the most well known ones are wavelets approaches [1], stochastic approaches [2], Partial Different Equations (PDEs) and variational approaches [3–5]. Readers are invited to refer [6] and references therein for details on the subject matter.

In quite a few imaging applications which are using coherent imaging techniques, we come across multiplicative noise [7]. Synthetic aperture radar (SAR), ultrasound (US) and positron emission tomography (PET) imaging are some of the imaging techniques where the data-dependent multiplicative noise severely disturbs the captured data. A radar sends a coherent wave, which is reflected on the ground and then registered by the radar sensor [8]. When the coherent wave gets reflected on a coarse surface (compared to the radar wavelength), the image processed by the system is degraded by a large amplitude noisy signal, this causes a specked image [7]. Since speckle is a data-dependent noise, the noise distribution varies with reference to the intensity characteristics of the image. In [8], the authors analyzed that, when the scatter density (the number of scatter per resolution cell) is more than 10, this speckle noise follows a Rayleigh distribution. However, in satellite images the final image is obtained by adding "n" images with independent intensity distributions and a common mean. Due to the aforementioned reasons the intensity follows a gamma law and therefore the speckle noise is also assumed to follow a gamma law with mean equals to one, refer [7] for further details. Therefore while denoising, one has to consider the distribution of the noise in order to perform a meaningful restoration. The probability density function (PDF) of the noise varies with

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regard to the applications under consideration. In SAR and US images the noise intensity(speckle noise) is observed to follow a Gamma distribution.

The data-dependent multiplicative noise is not analyzed quite extensively to the best of our knowledge. The first variational approach for handling such a kind of noise was proposed by Rudin et. al. (RLO model) in [9]. In this work the authors assume a data-dependent multiplicative noise that follows a Gaussian distribution with mean equal to one. But this model is of less practical interest, since the noise distributions in most imaging modalities do not follow a Gaussian model. There are few dedicated studies directed towards speckle reduction using anisotropic PDE models motivated by Perona–Malik model [4]. Among them, the famous ones are Speckle Reducing Anisotropic Diffusion (SRAD) [10] and Oriented-SRAD (OSRAD) [11]. A few modifications are done to SRAD, they include Detail preserving anisotropic diffusion (DPAD) [12], Improved SRAD (ISRAD) [13] and Speckle reducing self-snakes [14]. Besides these models, a relatively new variational model was proposed in [7] (AA-model). In this work the authors introduce a variational model with noise dependent fidelity characteristics. The diffusion term is borrowed form [3]. And the diffusion is well-defined in the space of bounded variations where the total variations are bounded. A few modifications were proposed for AA-model in the recent literature, readers are invited to refer [15,16] for further details. In addition to these variational models a log transformed model was used to re-represent a multiplicative model with an additive one, see [17] for details. Nevertheless, such a straight forward method does not lead to satisfactory results, because even in the log compressed images the noise does not become completely independent of data. Moreover, this formulation results in non-preservation of average gray-levels in the processed image, refer to [7] for further details.

Rest of the paper is organized as follows. In Section 2 multiplicative noise models are reviewed. In Section 3 the proposed model, its mathematical formulations and characteristics are discussed in detail. Numerical implementations of the proposed PDE is highlighted in Section 4. Experimental results and its discussions are done in Section 5. Finally the work is concluded in Section 6.

2. A review of image restoration under multiplicative noise setup

The multiplicative noise degradation model considered in many previous works follows the definition:

$$u_0 = un, \tag{1}$$

where u_0 is the observed image, u is the actual image and n is a multiplicative noise.

In general, image restoration models follow the variational form (written as a minimization problem):

$$\min_u \left\{ E(u) = \int_\Omega \phi(|\nabla u|)dxdy + \lambda \int_\Omega H(u)dxdy \right\}. \tag{2}$$

Here $\phi(|\nabla u|)$ is a regularization functional which derives to a diffusion term in the evolution PDE, where ∇u denotes the gradient of the image u. Here $H(u)$ denotes a fidelity term which ensures a minimum deviation to the restored image from the actual one. As the contribution of fidelity increases the extend of denoising reduces, and vice-versa. Therefore, a high contribution in terms of fidelity consequently reduces the extend of diffusion or the image is noticeably noisy, on the other hand a feeble contribution of fidelity results in over-smoothing of edges and other semantic features present in images. The control of fidelity and diffusion characteristics of the PDE is done using the regularization parameter λ.

Different regularization functionals are proposed in the literature for image restoration. In [3] the authors introduce the functional $\phi(u) = |\nabla u|_{TV}$ (here we note that $|.|_{TV}$ denotes the total variation norm of the functional and $|\cdot|$ stands for the "absolute" function). The TV norm of the functional u is defined as $TV(u) = \int_\Omega |\nabla u| dxdy$, where Ω is the two dimensional area of image support. In the above integration we use the variables $dxdy$ in place of $d\Omega$. Similarly in [4] the functional $\phi(u) = \mu^2/2 \log\left(1 + \frac{|\nabla u|^2}{\mu^2}\right)$ is used, where μ is a contrast parameter and in [18] the functional $\phi(u) = \mu^2\left(\sqrt{(1 + \frac{|\nabla u|^2}{\mu^2})} - 1\right)$ (where μ denotes the contrast parameter) is proposed for regularization. Further we note that in the models [4,18] the fidelity term is assumed to be zero.

In [9] a variational restoration model (RLO model) is proposed for general multiplicative noise distributions (however it is best suited for Gaussian multiplicative noise), with unit mean and noise variance σ^2. The energy minimization problem considered in this work can be read as:

$$\min_{u \in BV(\Omega)\cap L^2(\Omega)} \left\{ E(u) = \int_\Omega |\nabla u| dxdy + \lambda/2 \int_\Omega \left(\frac{u_0^2}{u^2} - 1\right) dxdy + \mu \int_\Omega \frac{u_0}{u} dxdy \right\}, \tag{3}$$

where $|\Omega|$ denotes the total pixels in the image area, u and u_0 are the original and observed images, respectively and σ^2 is the noise variance. Here $\int_\Omega |\nabla u| dxdxy$ denotes the TV norm of the functional. The Gradient descent solution amounts to:

$$u_t = div \frac{\nabla u}{|\nabla u|} - \lambda \frac{u_0^2}{u^3} - \mu \frac{u_0}{u^2}, \tag{4}$$

where div stands for divergence operator, the PDE in the above equations follow the boundary condition (Neumann boundary)