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Turbulent dispersion in the ocean

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Abstract

The mathematical framework for turbulent transport in the ocean is reasonably well established. It may be applied to large-scale fields of scalars in the ocean and to the instantaneous or continuous discharge from a point. The theory and its physical basis can also provide an interpretation of passive scalar spectra. Spatial variations in the rate of turbulent transfer can be related to the movement of the center of mass of a scalar and to a formulation in terms of entrainment. The relative dispersion of a scalar with respect to its center of mass and the streakiness of the concentration field within the relative dispersion domain need to be considered. In many of these problems it is valuable to think in terms of simple models for individual streaks, as well as overall statistical properties.

Keywords: Turbulent dispersion; Ocean mixing; Ocean eddies

1. Introduction

Including the effects of processes that are unresolved in models is one of the central problems in oceanography. In particular, for temperature, salinity, or some other scalar, one seeks to parameterize the eddy flux in terms of quantities that are resolved by the models. This has been much discussed, with determinations of the correct parameterization relying on a combination of deductions from the large-scale models, observations of the eddy fluxes or associated quantities, and the development of an understanding of the processes responsible for the fluxes. The key remark to make is that it is only through process studies that we can reach an understanding leading to formulae that are valid in changing conditions, rather than just having numerical values which may only be valid in present conditions.

Rather than attempt a comprehensive review, this brief article will summarize, as simply as possible, some basic ideas and results on dispersion in a turbulent flow, drawing attention in particular to results that may go beyond standard texts, such as that of Csanady (1973). Some fundamental fluid dynamical ideas and their application to the ocean will be described in Section 2. Quite apart from the importance of turbulent dispersion for the evolution of large-scale patterns in the ocean, it also determines the concentration of material released from a point source, either instantaneously or continuously. This will also be reviewed.

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For an instantaneous release, it is important to consider not only the "absolute" dispersion with respect to the point of release, but also the "relative" dispersion with respect to the center of mass of the released substance (e.g. Csanady, 1973; Fischer et al., 1979; Bennett, 2006). This will be reviewed in Section 3. Furthermore, the "streakiness" within the domain of relative dispersion may be a matter of concern and will be discussed.

The connection between the flux of a substance, with sharp gradients ultimately disappearing by molecular diffusion, and the adiabatic stirring associated with the dispersion of marked particles, will be reviewed in Section 4. In particular, standard ideas on the connections between stirring and mixing have been generalized to allow for the treatment of a hierarchy of different turbulent motions. Other, non-turbulent, mechanisms for dispersion in the ocean will be mentioned in Section 5, though not reviewed in detail.

Although eddy fluxes of potential vorticity or other dynamical quantities are also carried by particles, this paper will be concerned only with scalars. I hope that the non-expert reader will find it a useful introduction and that the expert reader will find one or two items of interest to compensate for shortcomings.

2. Eddy fluxes

We consider an ocean in which some scalar has concentration $C = \overline{C} + C'$ where $\overline{C}$ is the ensemble average of C and C' is its fluctuation. In practice the ensemble average is replaced by an average over time or space. This requires that there be a spectral gap, i.e. a band of frequency or wavenumber with little variance, between the slowly varying mean and the rapidly varying fluctuations. This assumption may well be hard to justify; we return to it later. The equation for the evolution of the mean state $\overline{C}$ involves the eddy flux $\mathbf{F} = \overline{\mathbf{u}C'}$, where $\mathbf{u}$ is the velocity fluctuation.

It is well recognized that $\mathbf{F}$ need not be aligned with the local gradient $\nabla\overline{C}$, but may be written in tensor form as

$$F_i = -T_{ij}\frac{\partial\overline{C}}{\partial x_j}. \tag{1}$$

This is formally possible for any flux, but the connection to the local mean gradient may only make physical sense if the motions responsible for the flux have a "mixing length" that is small compared with the distance over which $\overline{C}$ varies significantly.

We may write $T_{ij} = K_{ij} + S_{ij}$ where $K_{ij} = \frac{1}{2}(T_{ij} + T_{ji})$ and $S_{ij} = \frac{1}{2}(T_{ij} - T_{ji})$. The symmetric tensor K_{ij} is diagonalizable and is likely to represent down-gradient diffusion parallel to the principle axes of the tensor. We return to this later. The antisymmetric tensor S_{ij} has an associated "skew flux" $\mathbf{F}_s$ given by

$$\mathbf{F}_{si} = -S_{ij}\frac{\partial\overline{C}}{\partial x_j} = -(\mathbf{D}\times\nabla\overline{C})_i, \tag{2}$$

where $\mathbf{D} = -(S_{23}, S_{31}, S_{12})$. This flux is perpendicular to $\nabla\overline{C}$ and may be written as

$$\mathbf{F}_s = -(\nabla\times\mathbf{D})\overline{C} + \nabla\times(\mathbf{D}\overline{C}). \tag{3}$$

The second term of this flux is non-divergent and so does not affect the evolution of $\overline{C}$. The first term represents advection of $\overline{C}$ with a velocity $\mathbf{U}_s = -(\nabla\times\mathbf{D})$ which may be written as

$$\mathbf{U}_{si} = \frac{\partial S_{ij}}{\partial x_j}. \tag{4}$$

This standard formalism (e.g. Rhines and Holland, 1977; Moffatt, 1983; Middleton and Loder, 1989) is purely kinematic. Further insights are obtained if we write the fluctuation C' in terms of a particle displacement $\mathbf{X}$ from the position where its value of C matches the local mean value. Then $C' = -X_j\partial\overline{C}/\partial x_j$ provided that $\mathbf{X}$ is small in magnitude compared with the distance over which $\nabla\overline{C}$ varies significantly. The eddy flux becomes

$$\overline{u_iC'} = -\overline{u_iX_j}\frac{\partial\overline{C}}{\partial x_j}. \tag{5}$$

The diffusivity K_{ij} is now $\frac{1}{2}(\overline{u_iX_j} + \overline{u_jX_i})$ and the antisymmetric tensor S_{ij} is given by $\frac{1}{2}(\overline{u_iX_j} - \overline{u_jX_i})$. The vector $\mathbf{D}$ may be written as $\frac{1}{2}\overline{\mathbf{X}\times\mathbf{u}}$ and the advection $\mathbf{U}_s$ from (4) may be written as $\mathbf{U}_{si} = \partial(\overline{u_iX_j} - K_{ij})/\partial x_j$.