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The pancyclicity and the Hamiltonian-connectivity of the generalized base-*b* hypercube

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Abstract

The interconnection network considered in this paper is the generalized base-b hypercube that is an attractive variance of the well-known hypercube. The generalized base-b hypercube is superior to the hypercube in many criteria, such as diameter, connectivity and fault diameter. In this paper, we study the Hamiltonian-connectivity and pancyclicity of the generalized base-b hypercube by the algorithmic approach. We show that a generalized base-b hypercube is Hamiltonian-connected for $b \geqslant 3$. That is, there exists a Hamiltonian path joining each pair of vertices in a generalized base-b hypercube for $b \geqslant 3$. We also show that a generalized base-b hypercube is pancyclic for $b \geqslant 3$. That is, it embeds cycles of all lengths ranging from 3 to the order of the graph for $b \geqslant 3$.

Keywords: Interconnection networks; Hypercubes; Pancyclicity; Hamiltonian-connectivity

1. Introduction

The Interconnection network is a mathematical model to represent the topology of a multiprocessor system. In massively parallel MIMD systems, the interconnection network plays a crucial role in issues such as communication performance, hardware cost, potentialities for efficient applications and fault tolerant capabilities [14]. An interconnection network is usually represented by a graph where the vertices represent the nodes and the edges represent the links.

Various interconnection networks are proposed, thus the portabilities of algorithms across these interconnection networks demonstrate considerable importance. That a host interconnection network can embed another guest interconnection network implies that the algorithms on the guest can be simulated on the host systematically. *Paths* and *cycles* are popular interconnection networks owing to their simple structures and low degrees. Moreover, many parallel algorithms have been devised on them [14–16]. Many literatures have

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discussed how to embed cycles and paths into various topologies [2,4,8–11,14]. A cycle with length s is denoted by $C(s)$, where $s \geqslant 3$. A graph is *Hamiltonian* if it embeds a Hamiltonian cycle that contains each vertex exactly once [6]. In other words, that a graph is Hamiltonian implies that it embeds the maximal cycle. However, in the *resource-allocated systems*, each vertex may be allocated with or without a resource [3,7]. Thus, it makes sense to discuss how to join a specific pair of vertices with a Hamiltonian path in such systems. For example, let X and Y be two vertices in a resource-allocated system, where the former and the latter are assigned with an input device and an output device, respectively. If we find a Hamiltonian path joining the pair of vertices, we can utilize the whole system to perform the systolic algorithm on a linear array [16]. A graph is *Hamiltonian-connected* if there is a Hamiltonian path joining each pair of vertices. No wonder that there are many researchers discussing the Hamiltonian-connectivity of various interconnection networks [5,17]. On the other hand, to execute a parallel program efficiently, the size of the allocated cycle must accord with the problem size of the program. Thus, many researchers study the problem of how to embed cycles of different sizes into an interconnection network. A graph is *pancyclic* if it embeds cycles of every length ranging from 3 to N, where N is the order of the graph [1].

The interconnection network considered in this paper is the n-dimensional *generalized base-b hypercube*, denoted by a $GH(b,n)$, which has been proved to possess many attractive properties, such as vertex-symmetry, edge-symmetry, efficient communication and high degree of fault tolerance [12,13,18,19]. To the best of our knowledge, there is no article addressing the above cycle embedding problems of the $GH(b,n)$. In this paper, we investigate the Hamiltonian-connectivity and the pancyclicity of the $GH(b,n)$.

The rest of this paper is organized as follows. In Section 2, we present some notations and background that will be used throughout this paper. In Section 3, we study the Hamiltonian-connectivity of the generalized base-b hypercube. In Section 4, we investigate the pancyclicity of the generalized base-b hypercube. Conclusions are given in Section 5.

2. Notations and background

For the definition of the generalized base-b hypercube, the *Cartesian product* of graphs is defined as follows.

Definition 1. Given two graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$, their Cartesian product, denoted by $G \times H$, is a graph (V_m, E_m), where $V_m = V_G \times V_H$ and $E_m = \{((p_1,\ q_1),(p_2,q_2)) | (p_1,q_1),(p_2,q_2) \in V_m$ and $((p_1 = p_2$ and $(q_1,q_2) \in E_H)$ or $(q_1 = q_2$ and $(p_1,p_2) \in E_G))\}$ [6].

Definition 2. The n-dimensional generalized base-b hypercube, the $GH(b,n)$, is defined recursively [13]:

1. A $GH(b,1)$ is a $K(b)$, where $K(b)$ is denoted for a complete graph with b vertices.
2. A $GH(b,n)$ is $GH(b,n-1) \times K(b)$ for $n \geqslant 2$.

That is, a $GH(b,n)$ is a $K(b)^n$. Clearly the $GH(b,n)$ is a generalization of the n-dimensional binary hypercube that is a $K(2)^n$. A $GH(b,n)$ comprises b^n vertices, each vertex x labeled by an n-digit number in radix b arithmetic $v_n\, v_{n-1} \ldots v_2 v_1$. The vertex x is connected to another vertex x' if and only if they differ by exactly one digit v_i, where $1 \leqslant i \leqslant n$. As illustrated in Fig. 1, the structure of a $GH(5,2)$ is shown. In this paper, the *outline graph* of a $GH(b,n)$, denoted by an $OG(GH(b,n))$, is to take each $v_n v_{n-1}\ \ldots\ v_2^*$ subnetwork as a supervertex; and a pair of supervertices V^* and U^* in the $OG(GH(b,n))$ is connected if and only if there exists a pair of vertices x_1 and x_2 in the $GH(b,n)$ such that x_1 and x_2 are connected, where x_1 in the V^* and x_2 in the U^*. Clearly, a pair of supervertices $V^* = v_n v_{n-1}\ \ldots\ v_2^*$ and $U^* = u_n u_{n-1}\ \ldots\ u_2^*$ is connected if and only if they differ by exactly one digit v_i, where $2 \leqslant i \leqslant n$. That is, if each $v_n v_{n-1}\ \ldots\ v_2^*$ subnetwork of a $GH(b,n)$ is taken as a supervertex, the $GH(b,n)$ will be transformed to a $GH(b,n-1)$. We have the following proposition.

Proposition 1. *An $OG(GH(b,n))$ is a $GH(b,n-1)$.*

In a supervertex $V^* = v_n v_{n-1} \ldots v_2^*$, the vertex $v_n v_{n-1} \ldots v_2 d_1$ is said to be the d_1 vertex of the V^*. By the structure of the $GH(b,n)$, vertex d of V^* and vertex d of U^* are connected if V^* and U^* are connected in the $OG(GH(b,n))$ for each $0 \leqslant d \leqslant b-1$. Clearly, if the $OG(GH(b,n))$ embeds a cycle of supervertices with length l,